Automatic Mesh Generation for Coastal Ocean Modeling: OceanMesh2D

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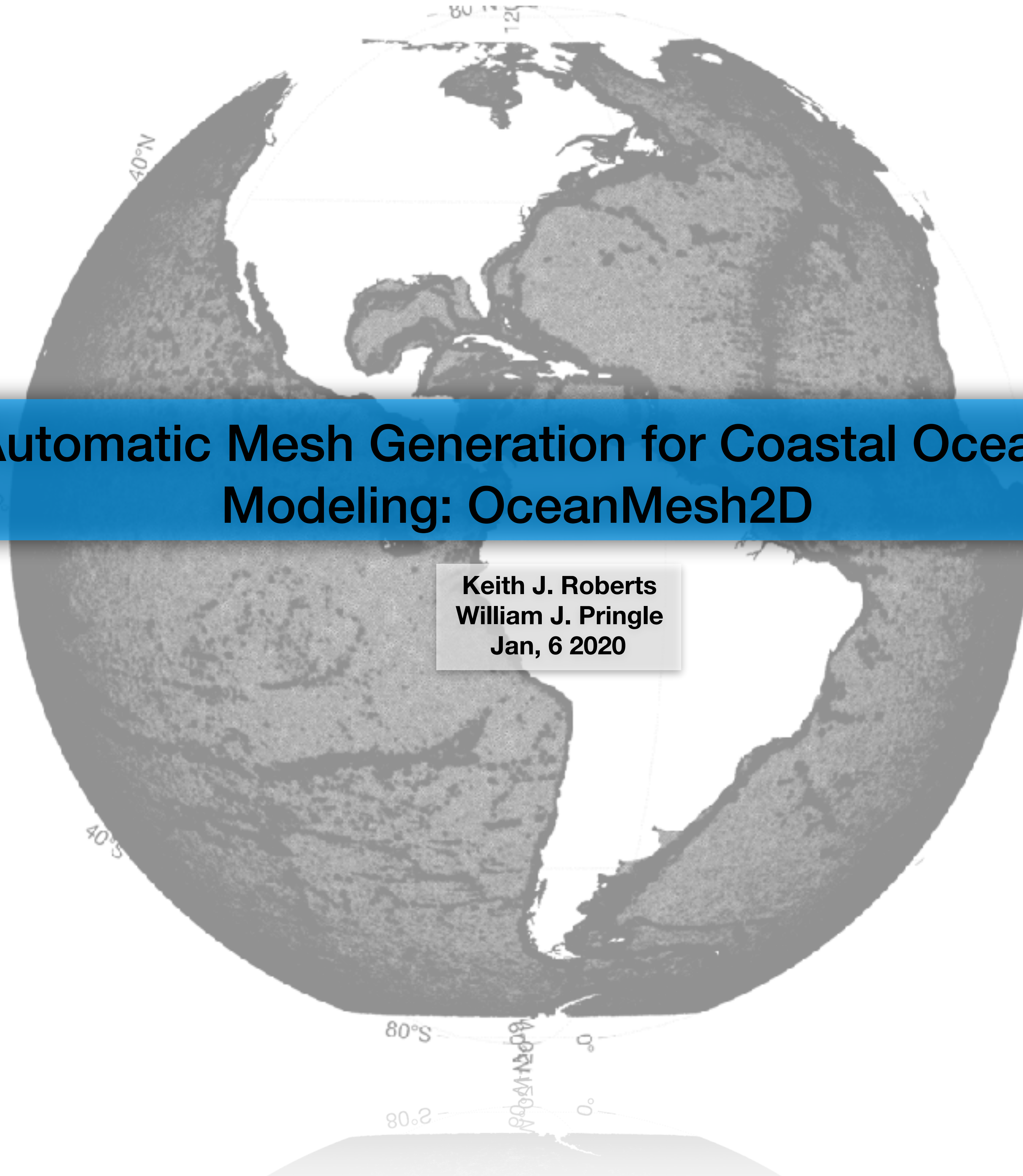
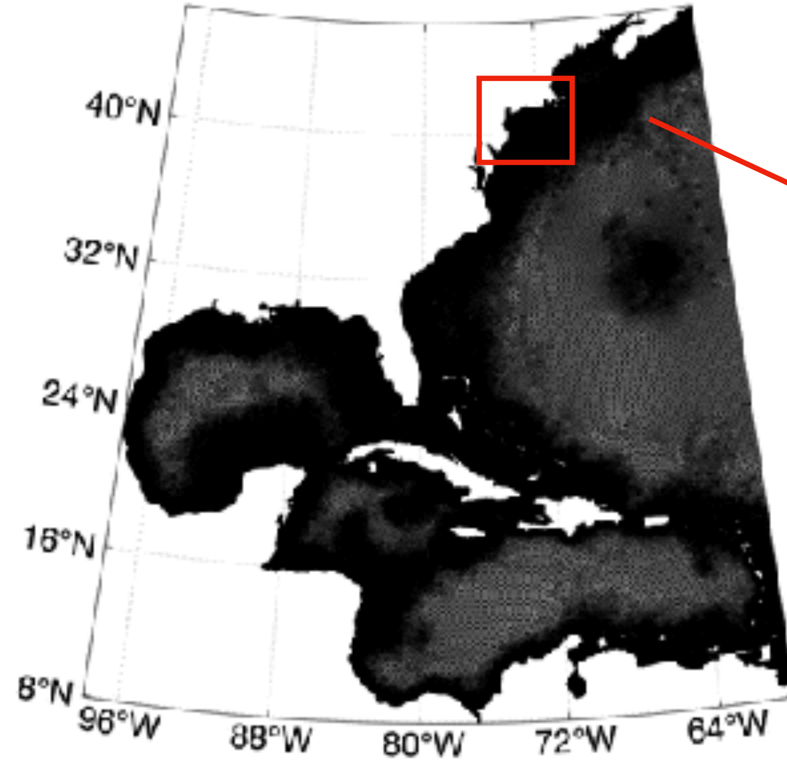
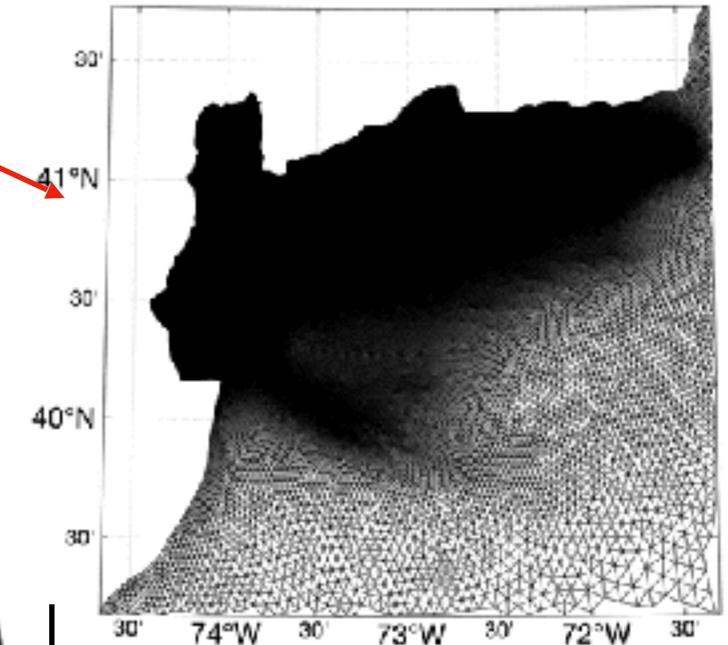
Jan, 6 2020

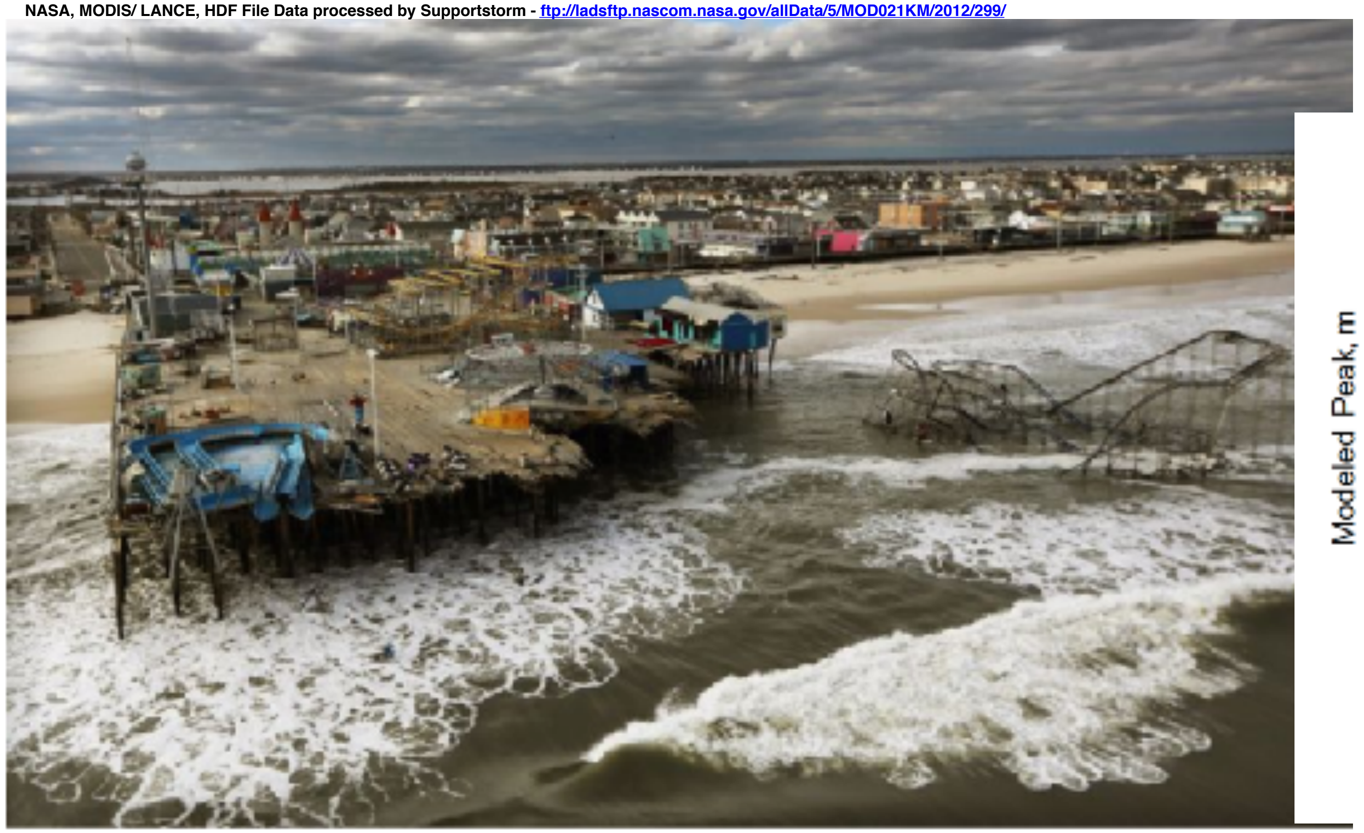


Physical processes & modeling

NASA, MODIS/ LANCE, HDF File Data processed by Supportstorm - ftp://ladsftp.nascom.nasa.gov/allData/5/MOD021KM/2012/299/

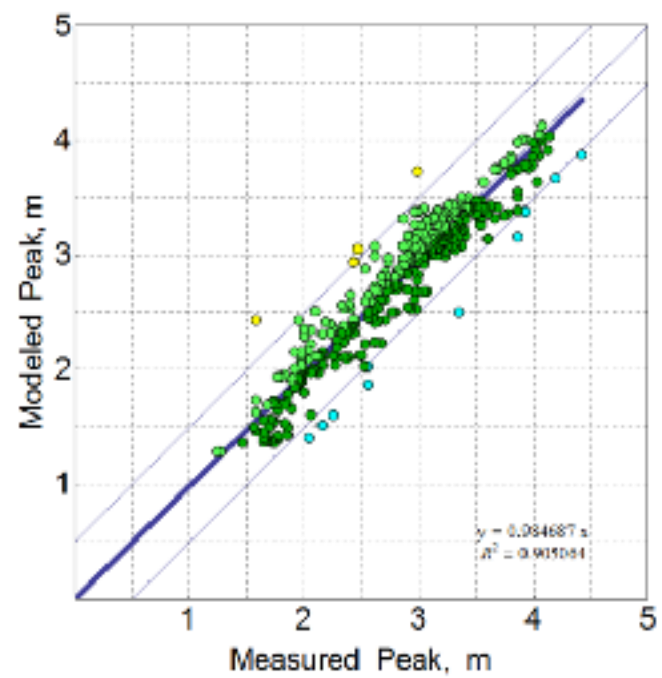


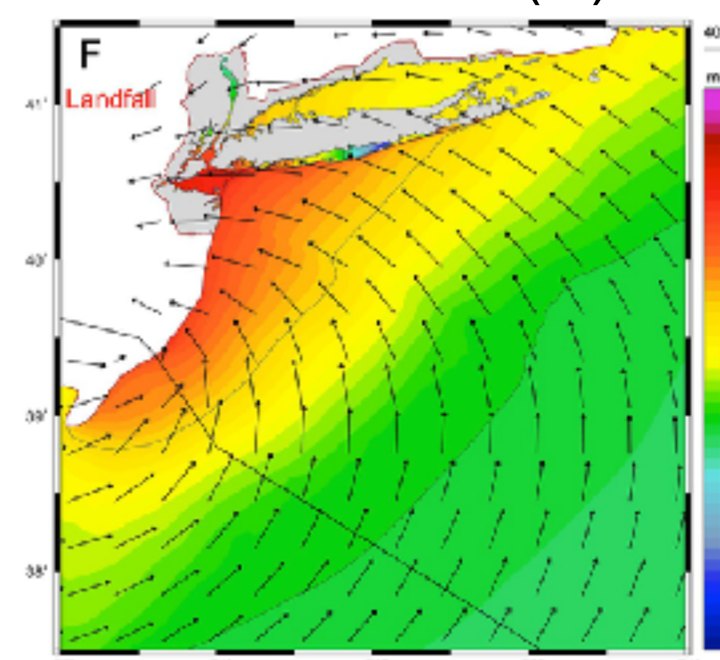




https://www.wired.com/story/what-weve-learned-about-climate-change-since-hurricane-sandy/

Predictions of water levels (m)





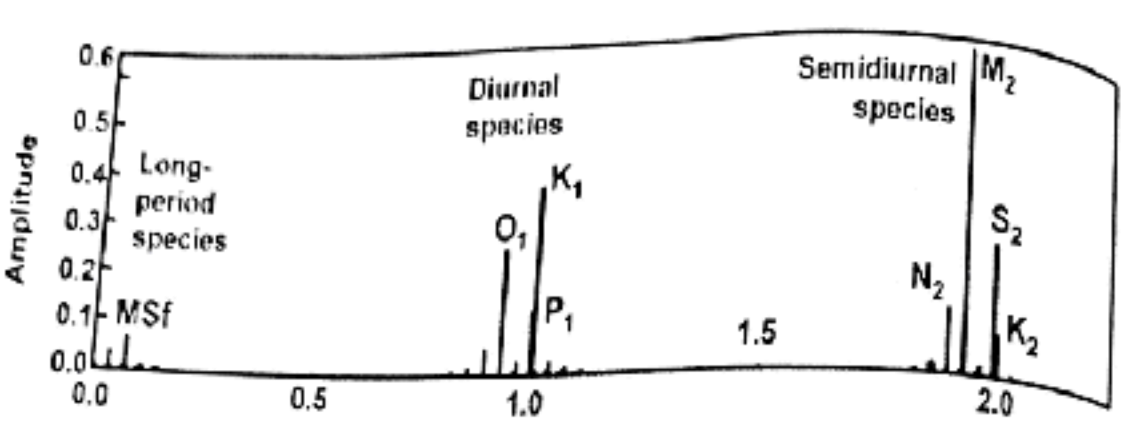
Hope et al. Hurricane Sandy (2012) Wind, Waves and Storm Surge in the New York Bight. I: Model Validation
Hope et al. Hurricane Sandy (2012) Wind, Waves and Storm Surge in the New York Bight. II: Analysis of Storm Surge Processes

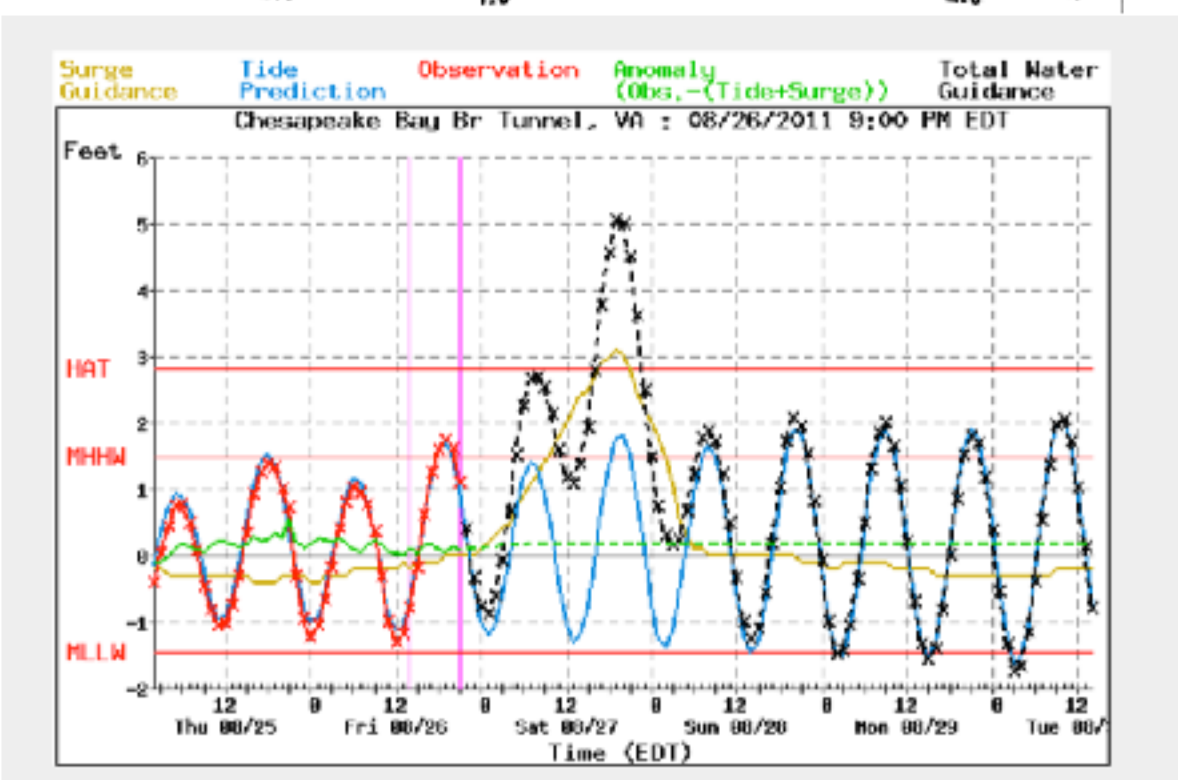
Used in simulations of surface tides

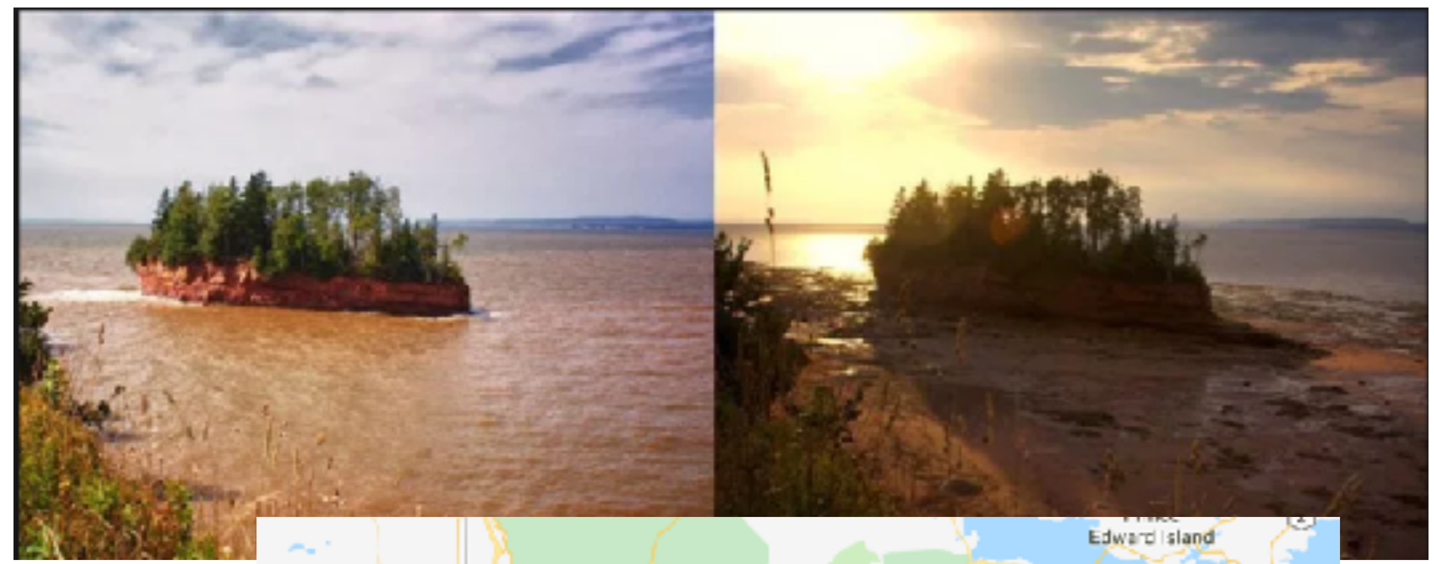
Celestial bodies (moon and sun) generate periodic variations in water levels called the tides due to gravitation.

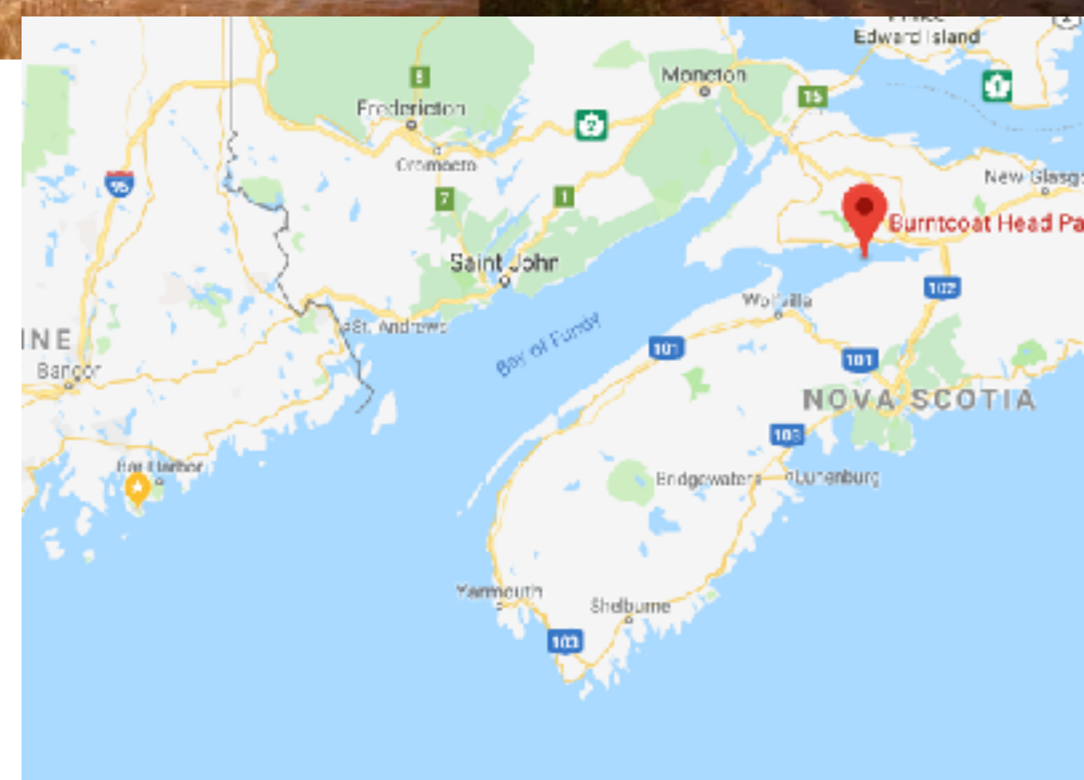
- Semi-diurnal driven primarily from moon.
- Diurnal driven by declination of moon w.r.t. to Earth's equatorial horizon.

Tide can vary by 45 ft (16-m) in Bay of Fundy





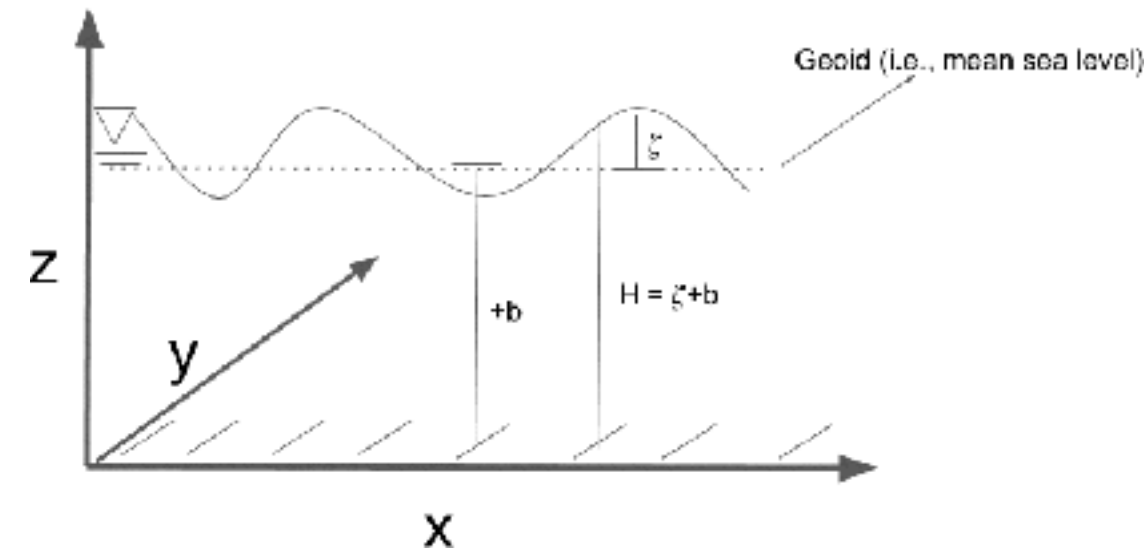




Governing Equations: “Shallow”-water equations

Key assumptions

Vertical scales << horizontal scales, “long waves”
Hydrostatic pressure



Primitive continuity equation

$$\frac{\partial \zeta}{\partial t} + \nabla \cdot (\mathbf{u}H) = 0 \quad \textbf{on} \quad \Omega$$

Depth-averaged momentum equations

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} + \tau_{bf}\mathbf{u} + f_c \hat{k} \times \mathbf{u} + g \nabla \zeta - \nu \nabla^2 \mathbf{u} = \mathbf{f} \quad \textbf{on} \quad \Omega$$

ζ	water surface elevation
$H \equiv \zeta + h_b$	total water column depth where h_b is the depth below the geoid.
$\mathbf{u}$	depth averaged velocity
$\mathbf{f}$	external body forces (e.g., winds)
$\tau_{bf}, f_c \hat{k}, g, \nu$	bottom friction term, Coriolis parameter, gravitational constant, kinematic viscosity

Ω computational domain

Manual mesh generation

1. **Trace shoreline boundaries from imagery.**
2. **Specify resolution zones by hand.**

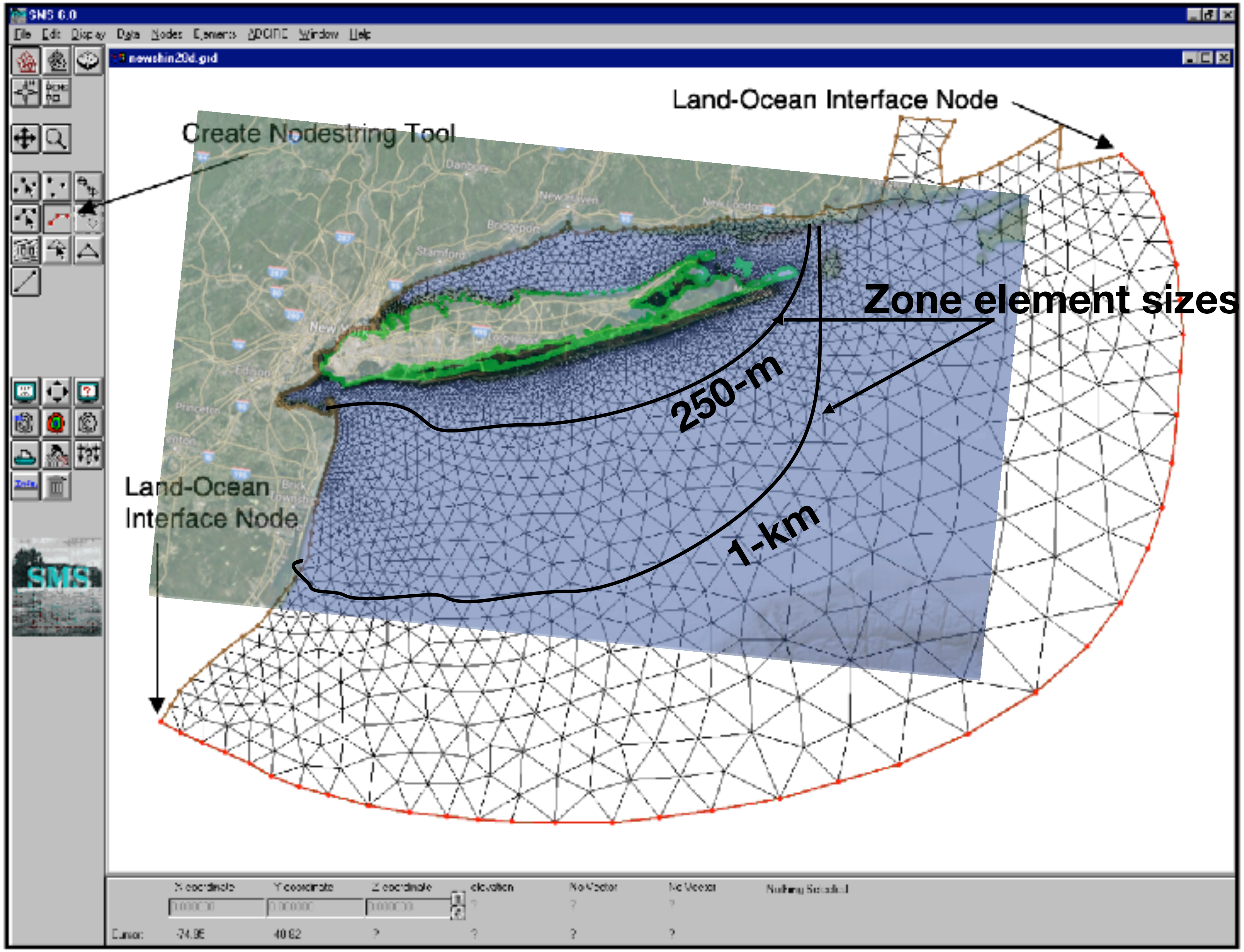
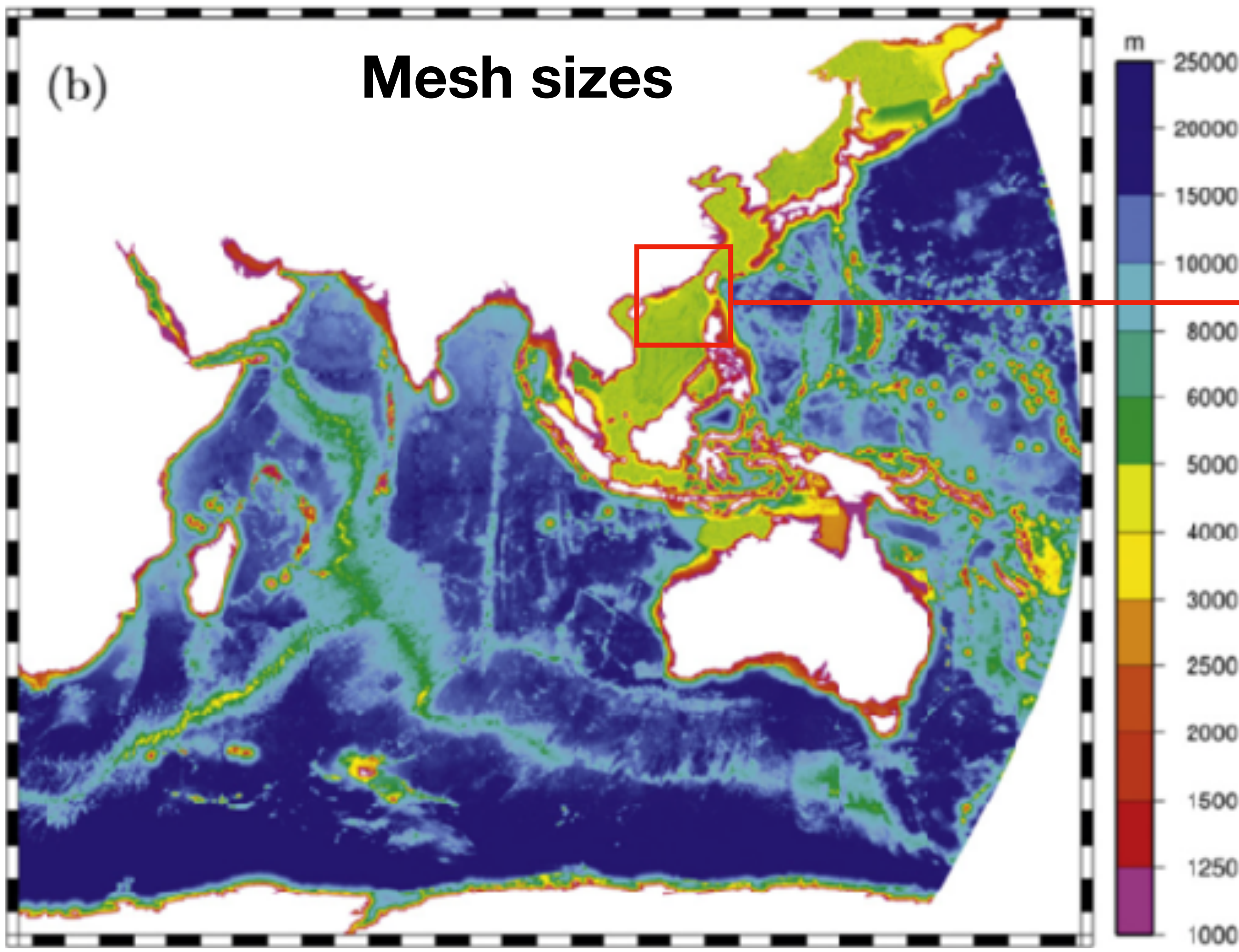


Figure 1. Graphical information for creating a nodestring on the ocean boundary

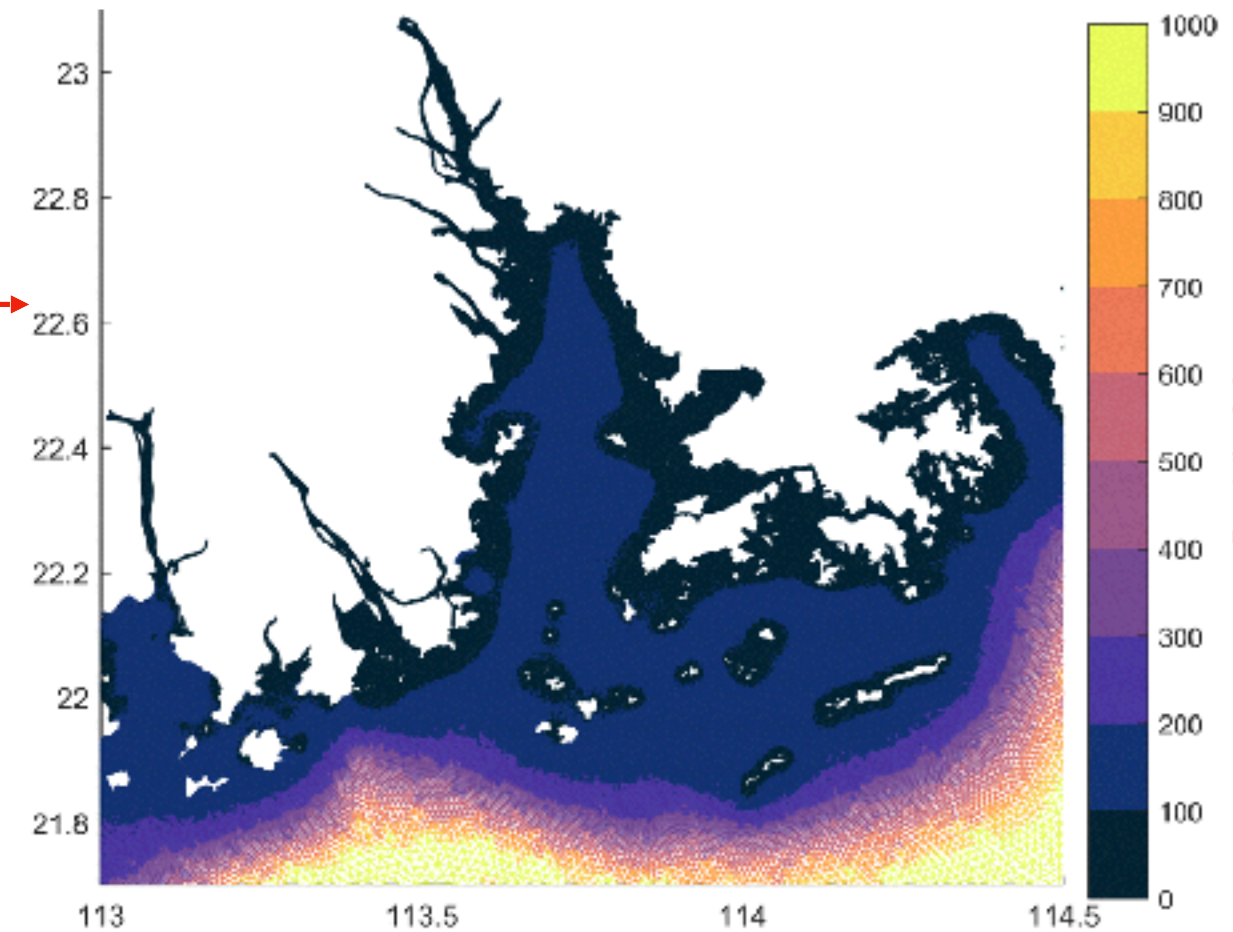
Zundel 2008, Surface-Water Modelling System (SMS) V8

Manual coastal ocean model generation with SMS…

7 years of work, 6 researchers! What took so long?



Mesh sizes around Hong Kong, China



Pringle et al. (2018). Finite-Element barotropic model for the Indian and Western Pacific Oceans: Tidal model-data comparisons and sensitivities. Ocean Modeling. Volume 129, September 2018, Pages 13-38

Irregular shoreline boundary

Shoreline is irregular and often contains features that are sub-grid scale at typical minimum resolution sizes used (25-100-m).

A simplification of boundary is required (often considered a pre-processing step).

Section of shoreline along northern Gulf Of Mexico

What is an efficient distribution of elemental resolution?

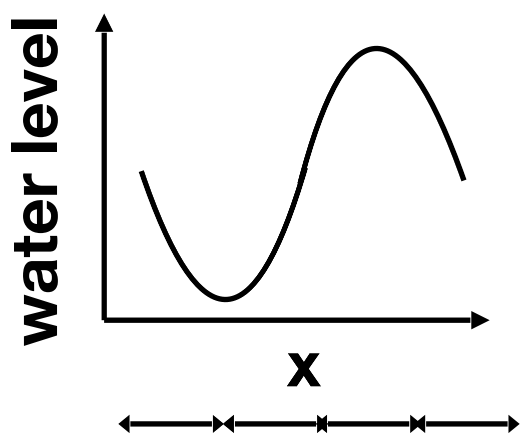
Vertex densities on the shelf are zoned according to the wavelength-to-gridscale mesh sizing heuristic.

Resolution transitions are slow $\approx \sqrt{H}$

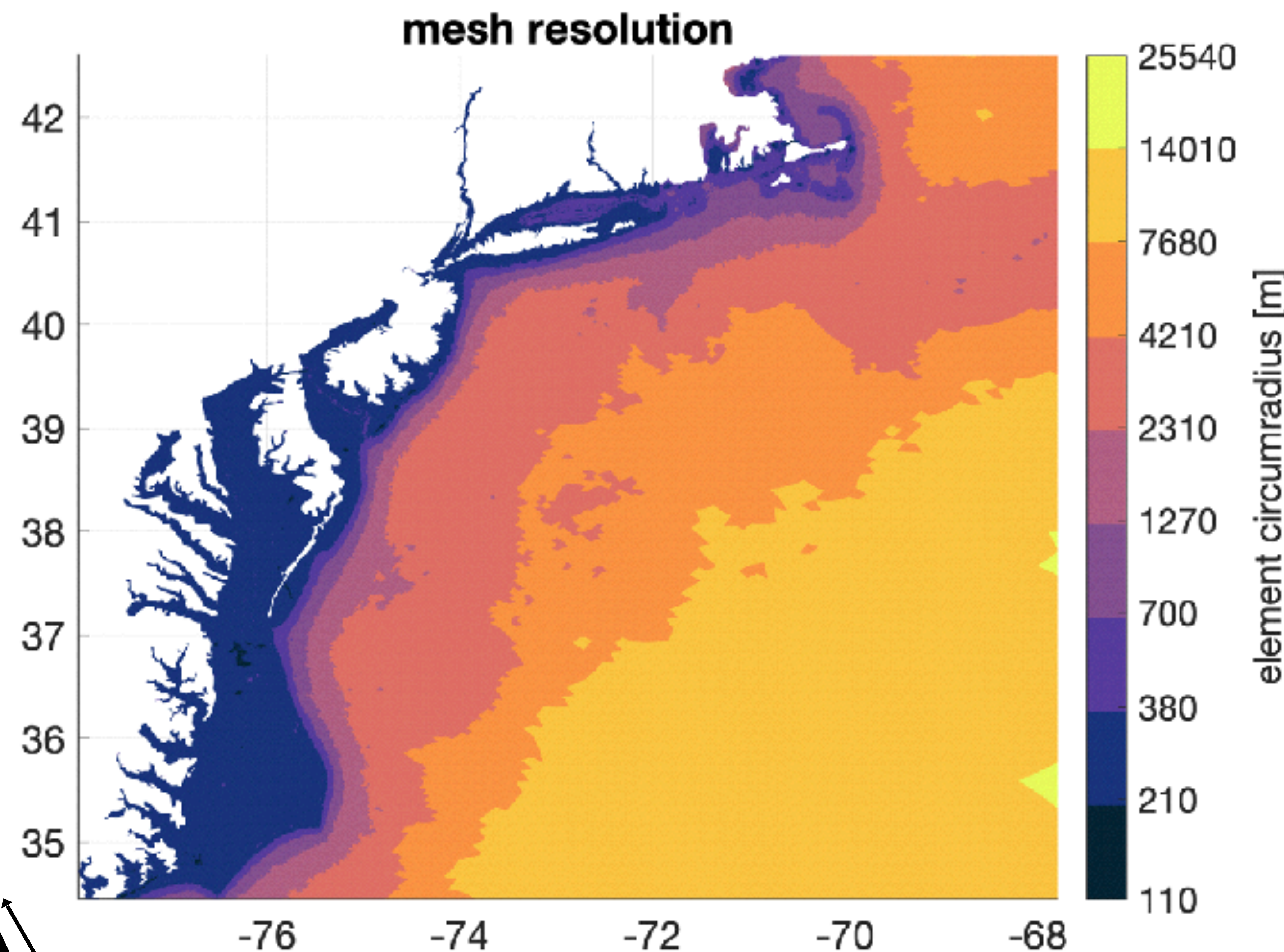
α_{wl} **is set artificially high (100+) to ensure shoreline resolution matches shelf resolution**

$$h = \frac{\lambda_{M2}}{\alpha_{wl}}$$

$$h = \frac{T_{M2}}{\alpha_{wl}}\sqrt{gH}$$



$\alpha_{wl} = 5$

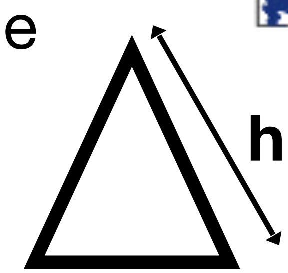


λ_{M2} wavelength of M2 tide

T_{M2} period of M2

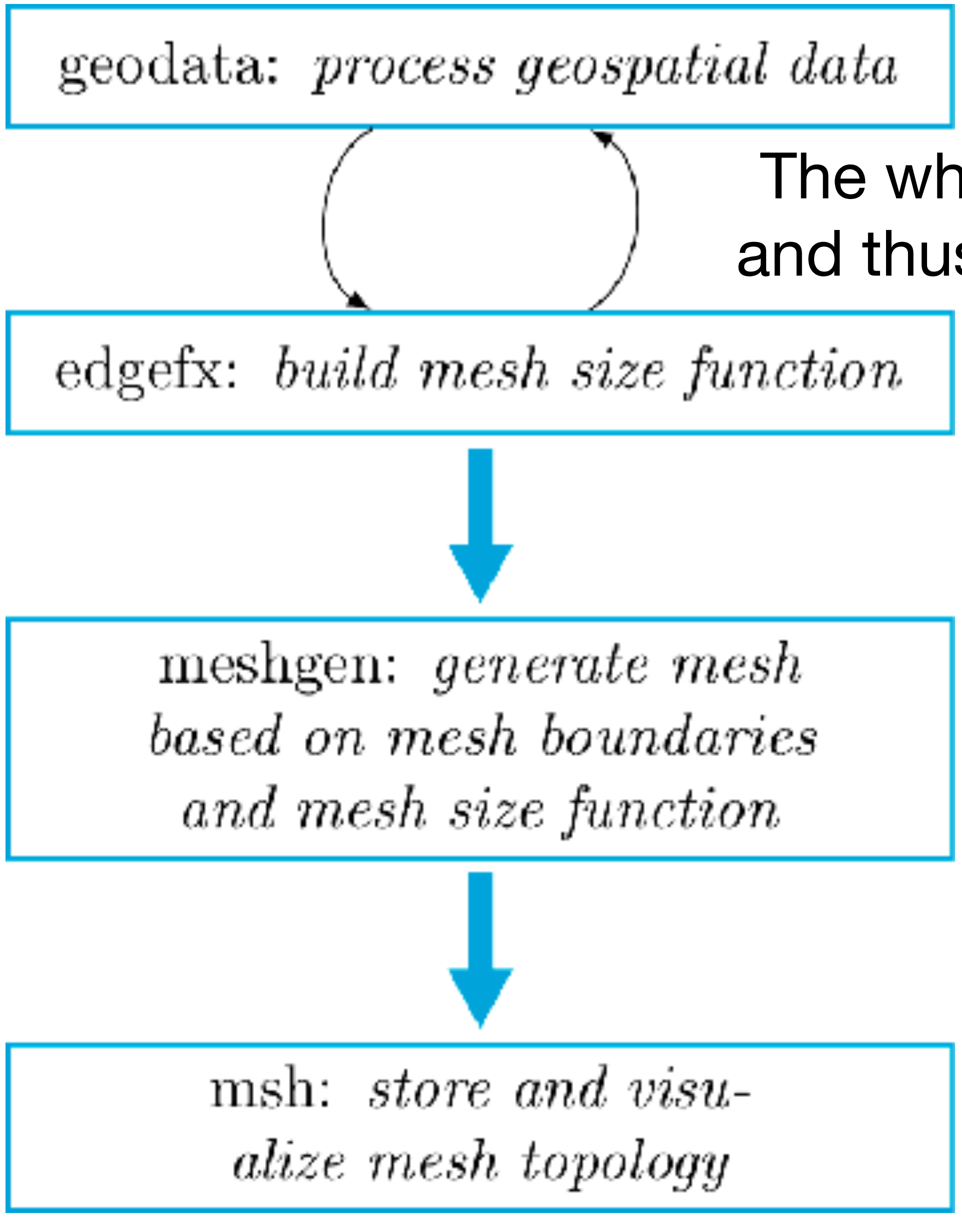
H depth below geoid

h edgelength of triangle



Automatic mesh generation: OceanMesh2D

An object orientated approach for coastal mesh development.



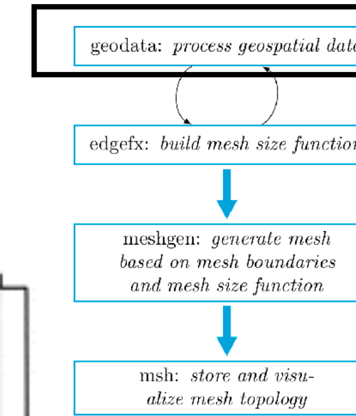
The whole process can be scripted and thus approximately reproducible

Software: https://github.com/CHLNDDEV/OceanMesh2D/tree/Projection

User-guide: Roberts, K. J., Pringle, W. J, 2018. OceanMesh2D: User guide - Precise distance-based two-dimensional automated mesh generation toolbox intended for coastal ocean/shallow water. https://doi.org/10.13140/RG.2.2.21840.61446/

Paper: Roberts, K. J., *et. al*. 2019. OceanMesh2D 1.0: MATLAB-based software for two-dimensional unstructured mesh generation in coastal ocean modeling, in press to *Geosci. Model Dev.*, https://doi.org/10.5194/gmd-2018-203

Geodata: shoreline is represented with signed-distance functions



Simplification of shoreline is not necessary beforehand

$$\Omega = S \cap bbox$$

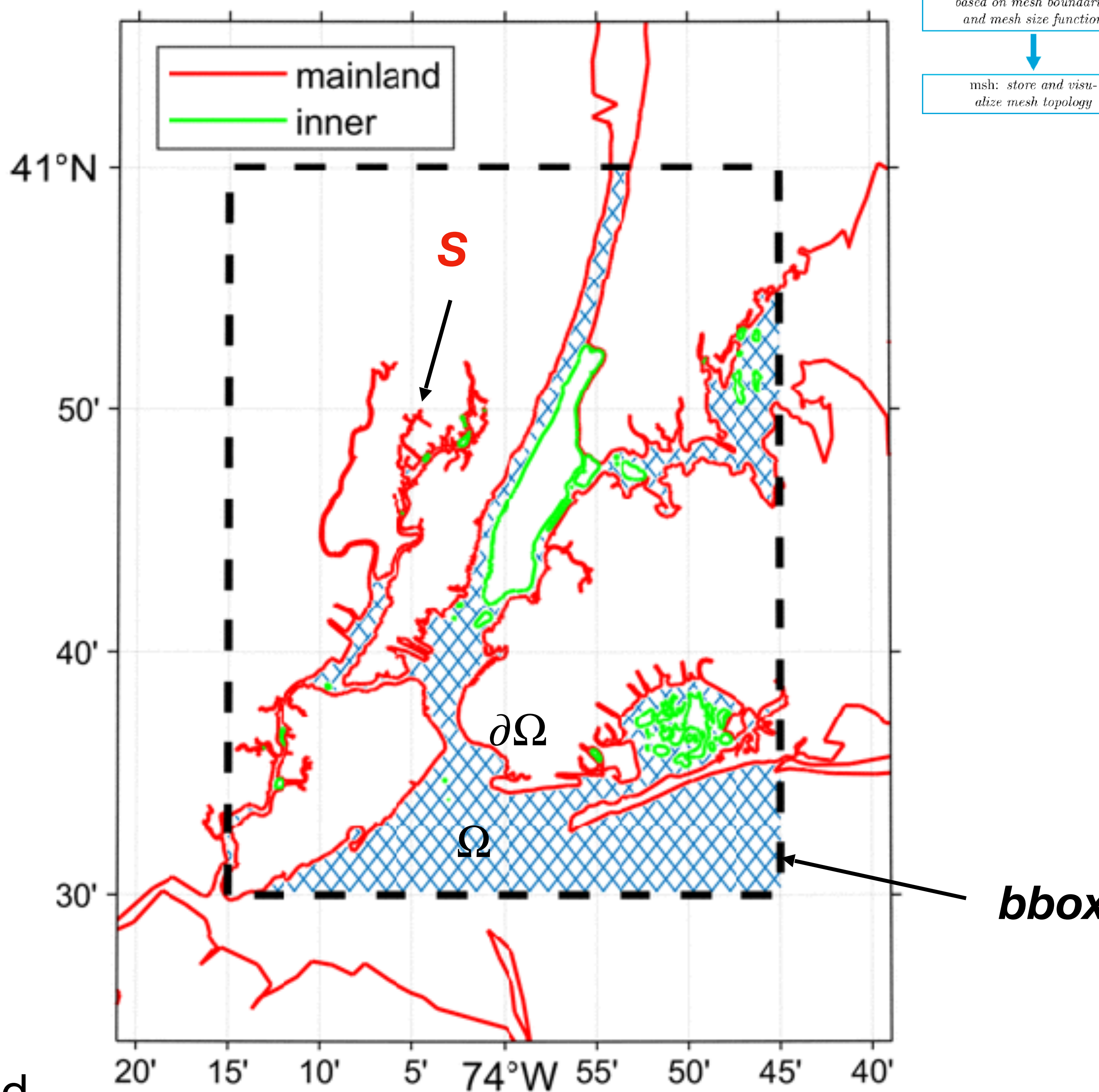
$$d(\boldsymbol{x})_\Omega = s_\Omega(\boldsymbol{x}) \min_{\boldsymbol{y} \subset \partial\Omega}(||\boldsymbol{x} - \boldsymbol{y}||)$$

sign function **nearest distance**

$$s(\boldsymbol{x})_\Omega := \begin{cases} -1, & \text{if } \boldsymbol{x} \in \Omega. \\ +1, & \text{if } \boldsymbol{x} \in \mathbb{R}^2 \setminus \Omega. \end{cases}$$

$$\Omega := \left\{ \boldsymbol{x} \in \mathbb{R}^2 : d(\boldsymbol{x})_\Omega \leq 0 \right.$$

$$\partial\Omega := \left\{ \boldsymbol{x} \in \mathbb{R}^2 : d(\boldsymbol{x})_\Omega = 0 \right.$$



*Shoreline are represented as piecewise linear segments at raw detail.

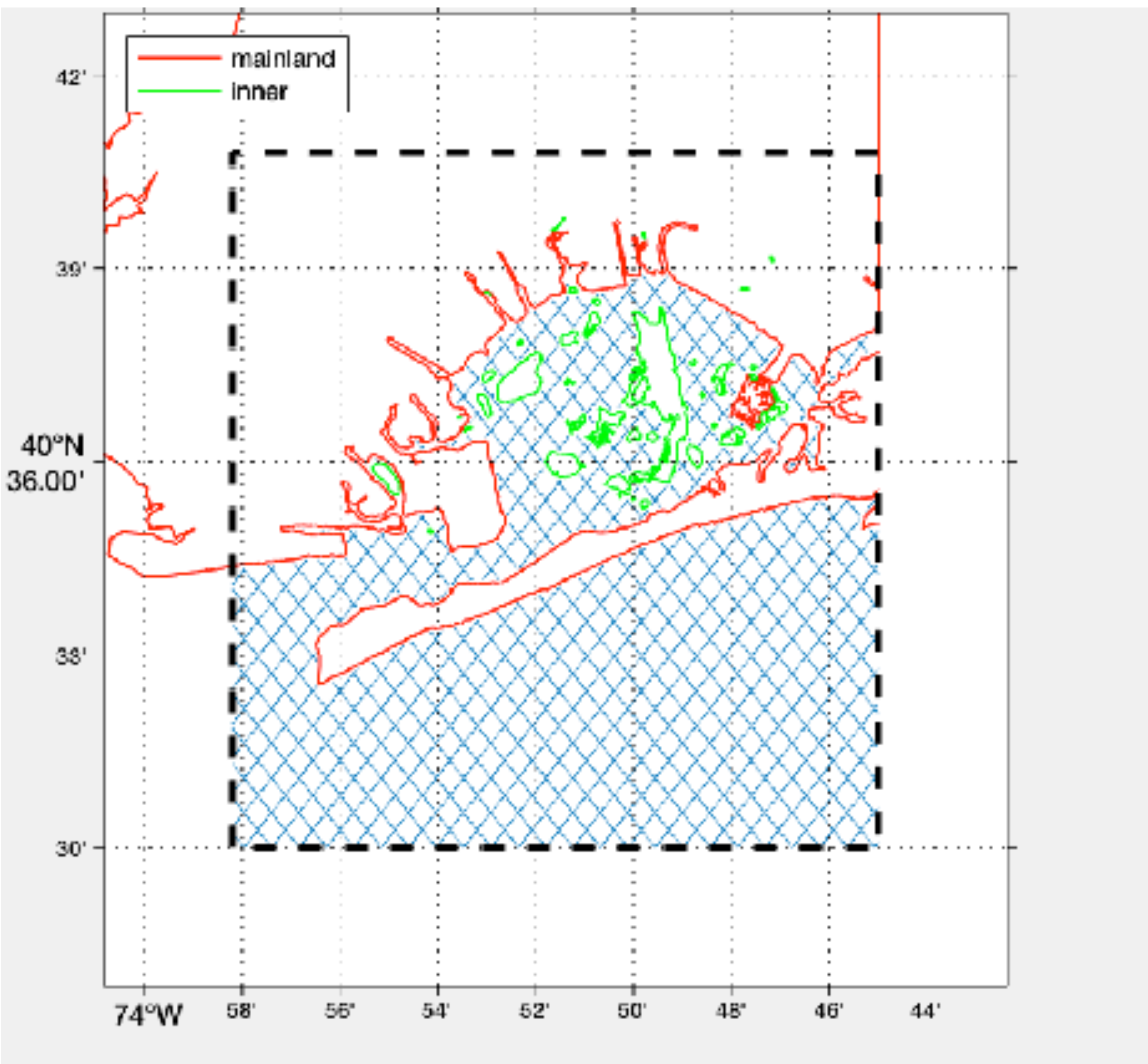
*Segments are classified by length and intersection with ***bbox***

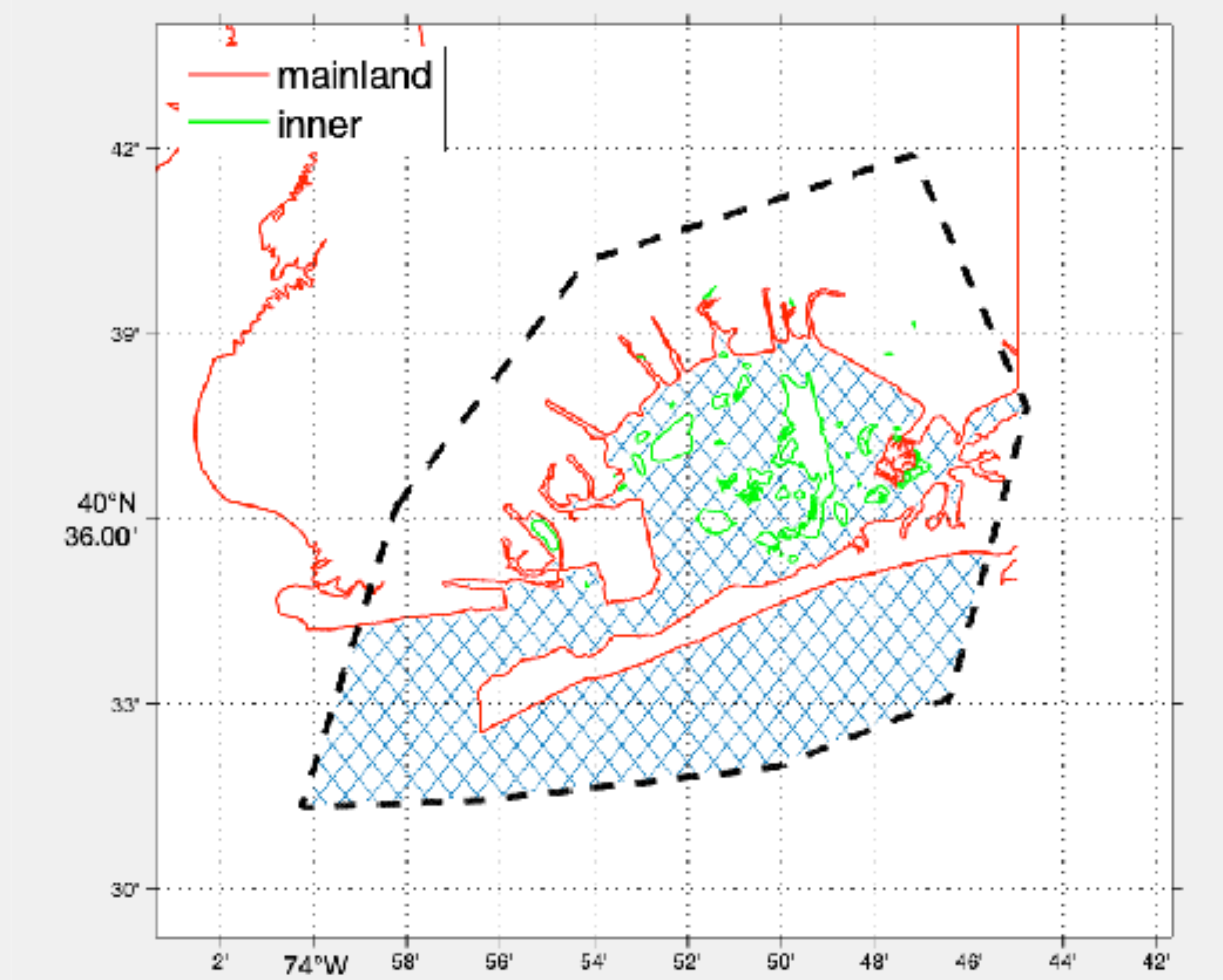
Geodata

Domains just have to be a polygon

bbox = [-73.9705   40.6032
-73.9028   40.6696
-73.7867   40.6983
-73.7463   40.6300
-73.7735   40.5517
-73.8351   40.5329
-73.9389   40.5240
-74.0040   40.5220
-73.9705   40.6032];

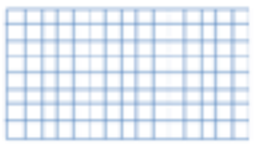
```
bbox      = [-73.97 -73.75  % lon_min lon_max
            40.5 40.68];  % lat_min lat_max
```

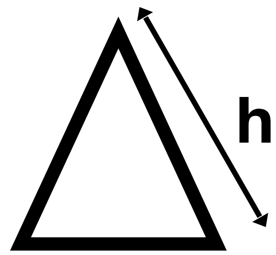




Edgefx: Mesh sizes are controlled via sizing functions

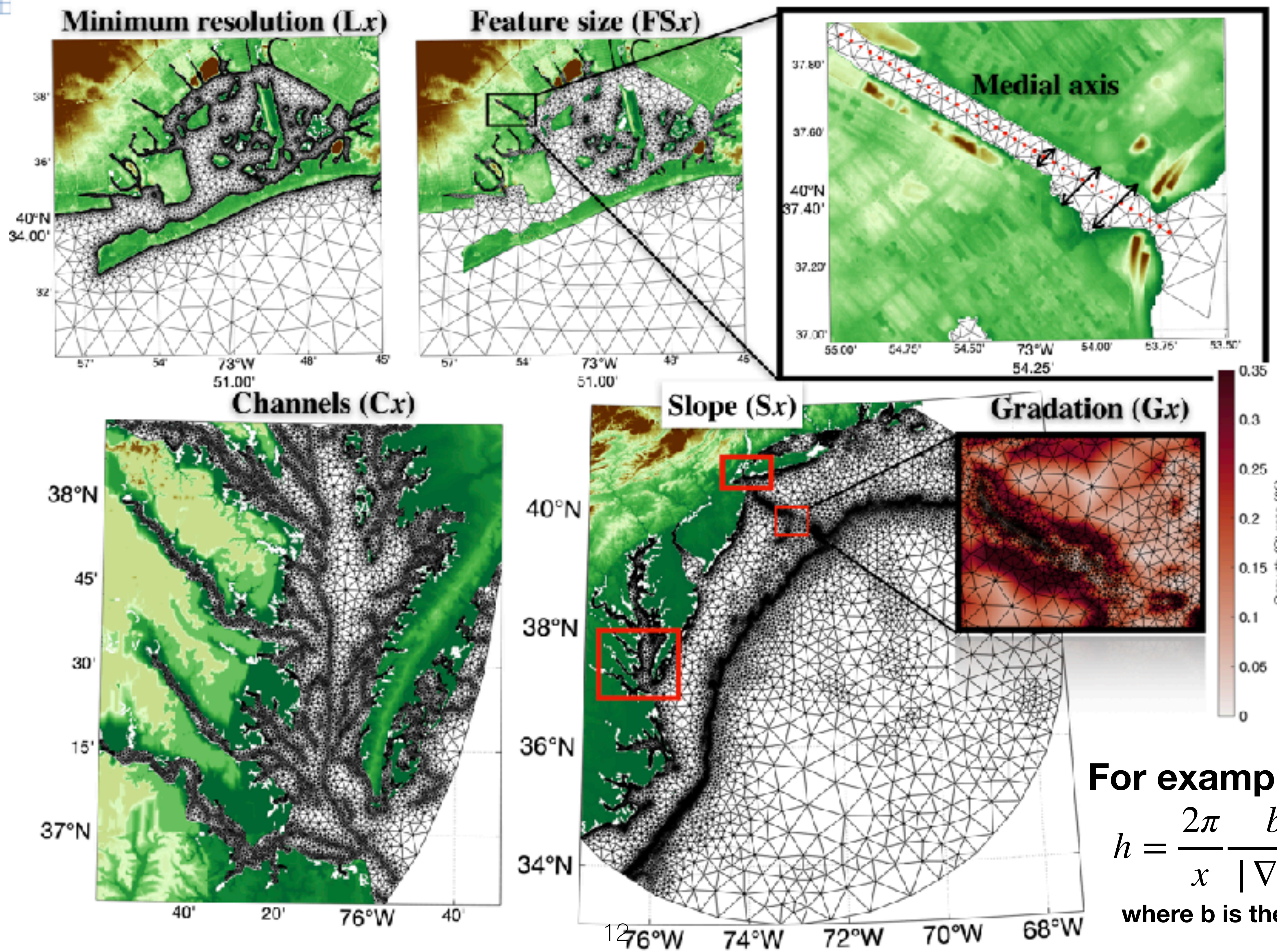
$h(x, y, z) \rightarrow h$





Combinations of mesh size functions are createds

$$h = min\left[(h_{Lx} \text{or} h_{Fsx}), h_{Sx}, h_{Cx}\right] \text{with} g \leq G$$



For example,

$$h = \frac{2\pi}{x} \frac{b}{|\nabla b|}$$

where b is the depth

Edgefx

Sizing fields can be built directly on DEM grids (and many at the same time)

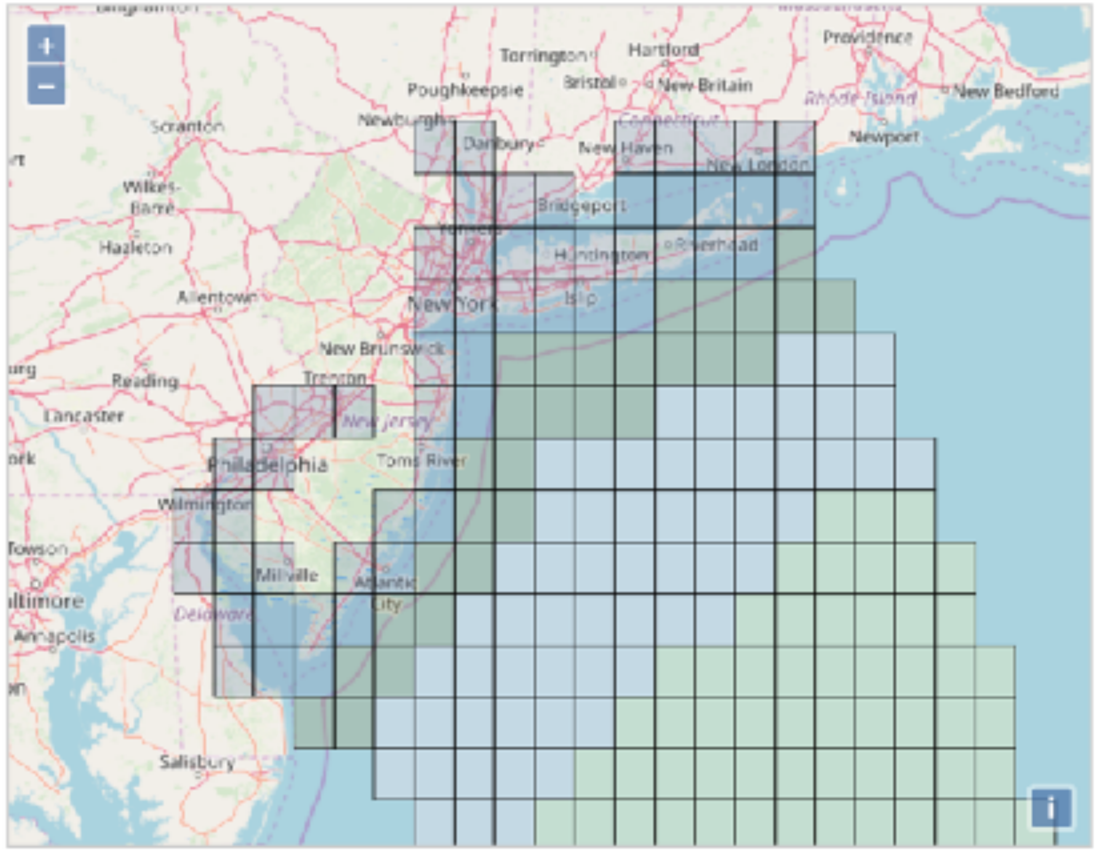
NCEI Hurricane Sandy Digital Elevation Models

NOAA's National Centers for Environmental Information (NCEI) is developing a suite of digital elevation models (DEMs) of the U.S. Atlantic Coast impacted by Hurricane Sandy in October 2012. These DEMs are the initial part of a planned framework for a seamless depiction of merged bathymetry and topography along U.S. coasts.

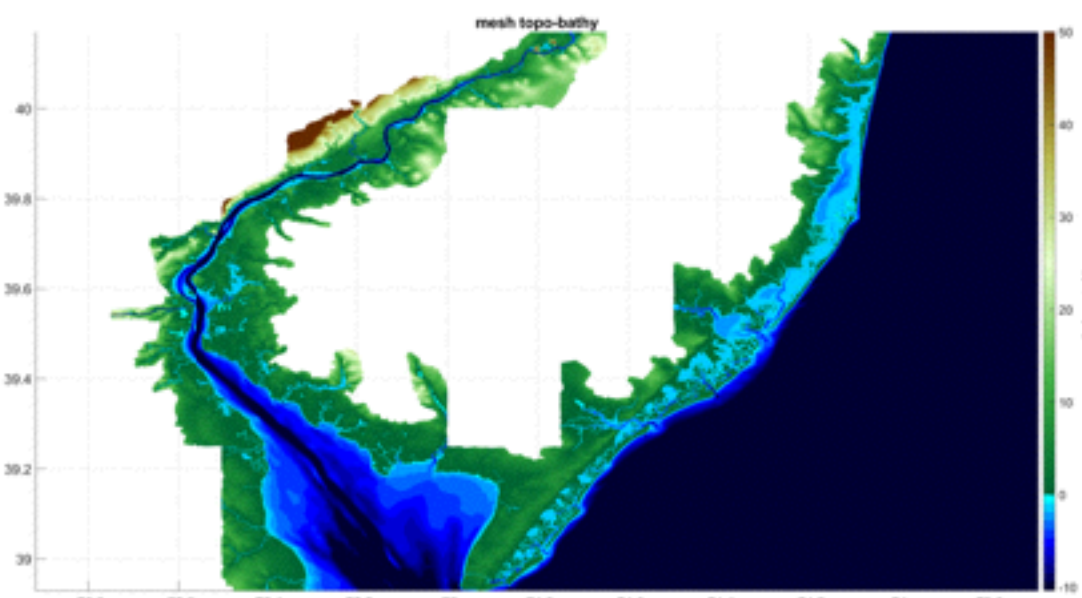
The DEMs telescope from the deep ocean floor to the coastal zone in 3, 1, 1/3, and 1/9 arc-second cell sizes. The 1/9 arc-second DEMs integrate both bathymetric and topographic data at the coast, while the offshore DEMs map bathymetry only.

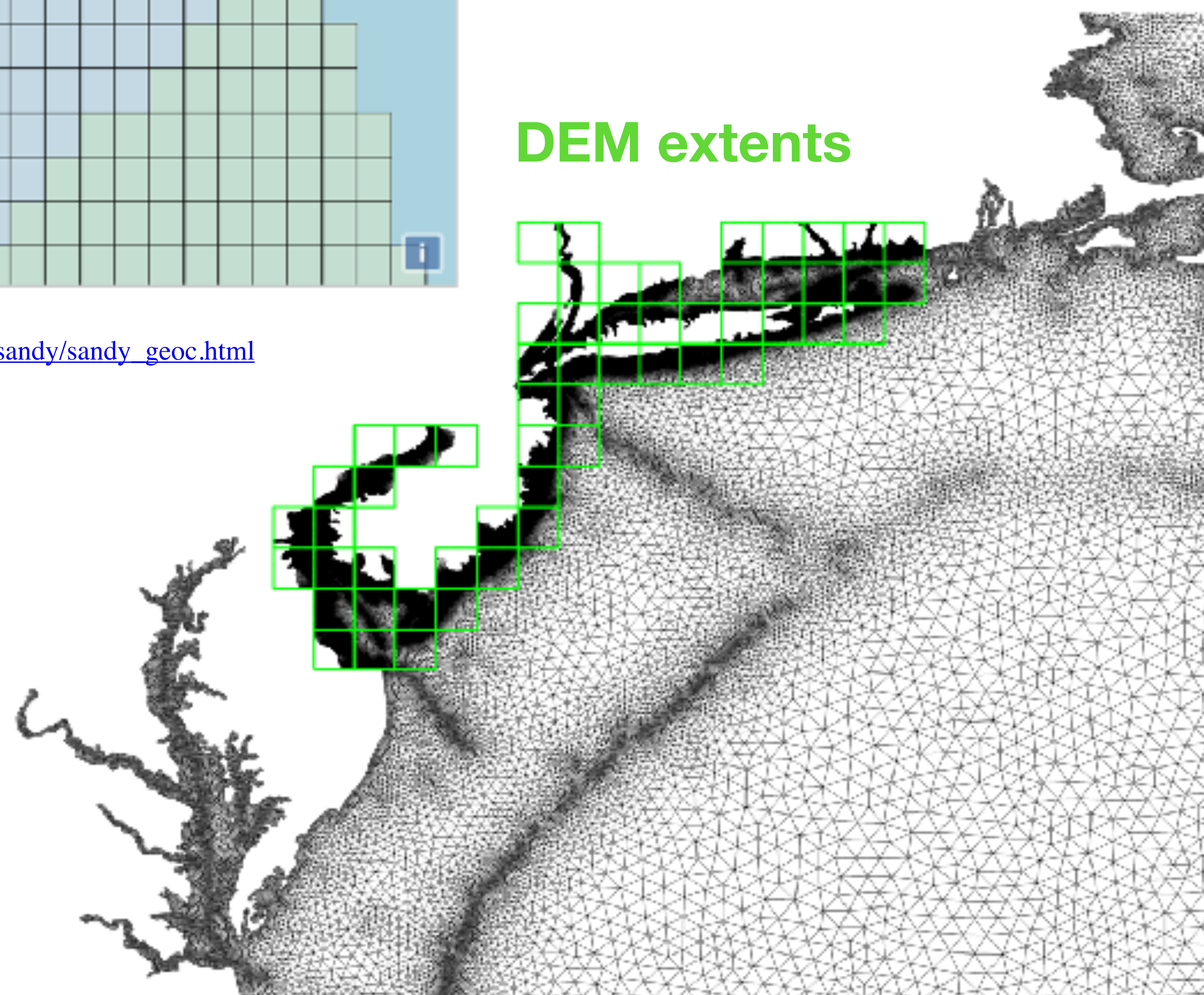
DEMs are tiled to enable targeted, rapid updates as new data become available.

Data Download:
Select region on map ➜
Access All Data
Access Metadata



https://www.ngdc.noaa.gov/mgg/inundation/sandy/sandy_geoc.html

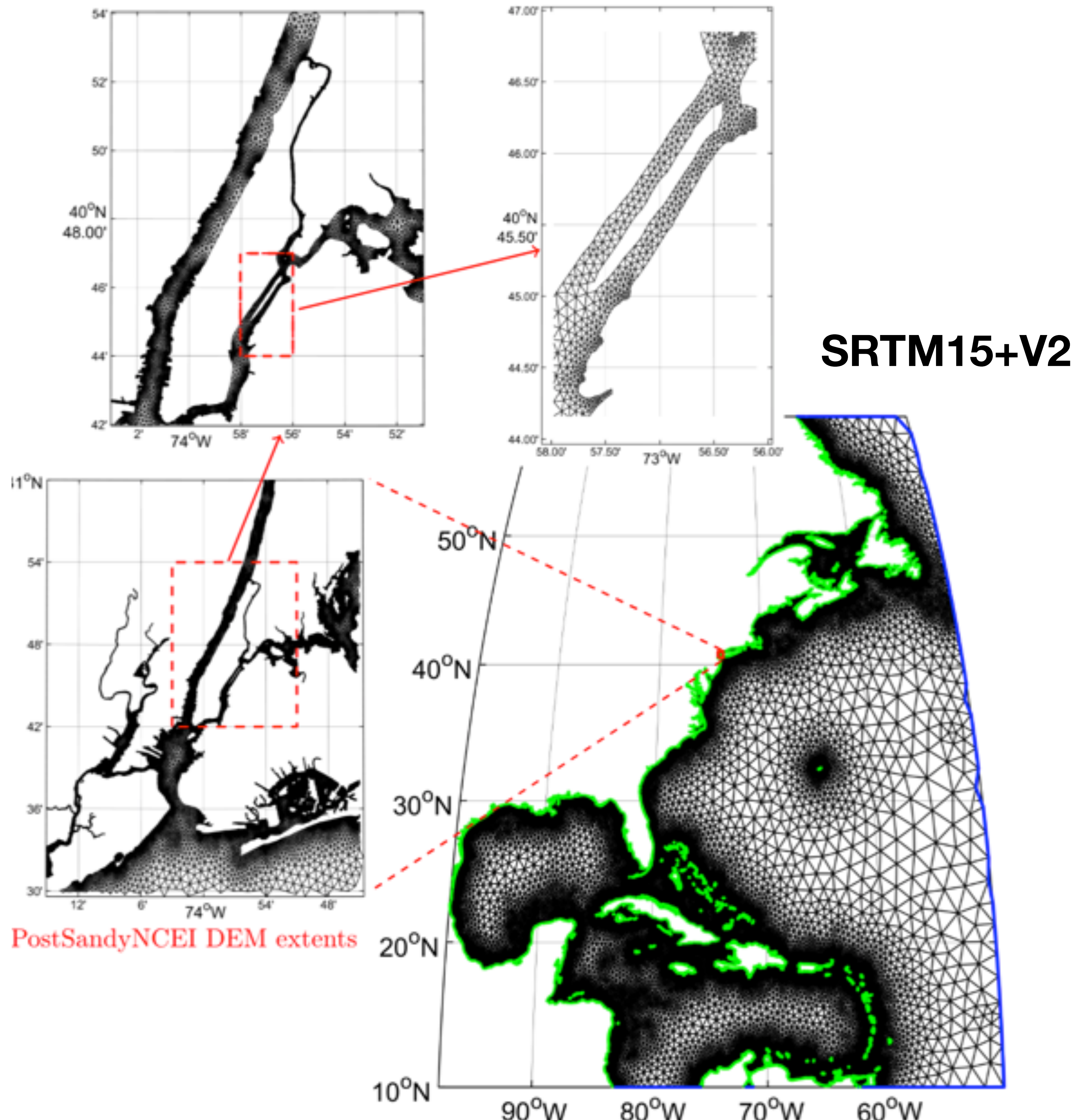




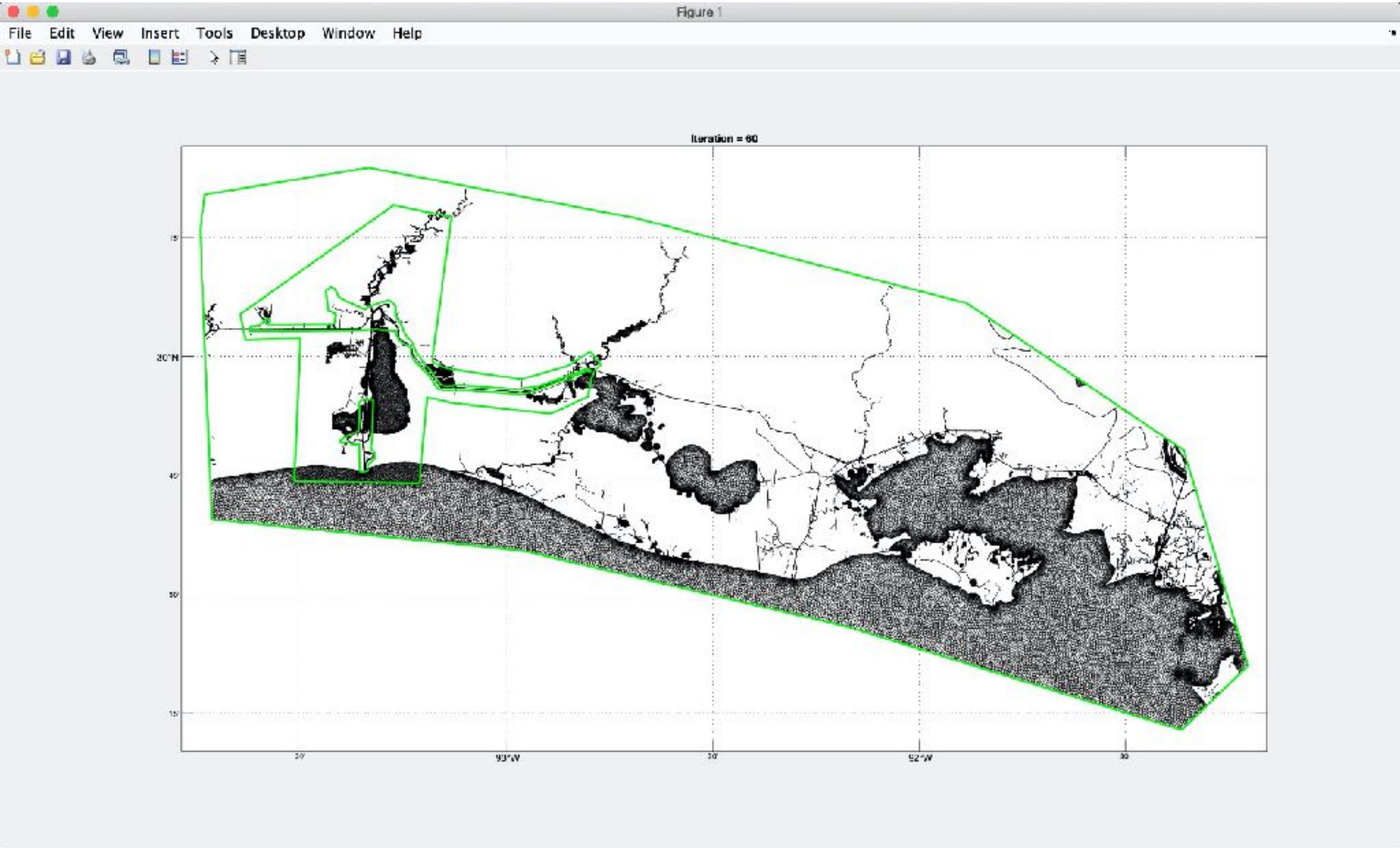
Multiscale meshing

Sizing fields can be nested with varying options to produce meshes with great element size variation.

```
%% STEP 1: set mesh extents and set parameters for mesh.
%% The greater US East Coast and Gulf of Mexico region

bbox = [-71.6 42.7; -64 30; -80 24; -85 38; -71.6 42.7]; %polygon boubox
min_el    = 1e3;                    % minimum resolution in meters.
max_el    = 50e3;                   % maximum resolution in meters.
wl        = 30;               % 60 elements resolve M2 wavelength.
dt        = 0;                % Automatically set timestep based on nearshore res
grade     = 0.35;             % mesh grade in decimal percent.
R         = 3;                      % Number of elements to resolve feature.

%% STEP 2: specify geographical datasets and process the geographical data
%% to be used later with other OceanMesh classes...
dem       = 'SRTM15+V2.nc';
coastline = 'GSHHS_f_L1';
gdat1 = geodata('shp',coastline,'dem',dem,'h0',min_el,...
                'bbox',bbox);

%% STEP 3: create an edge function class
fh1 = edgefx('geodata',gdat1,...
             'fs',R,'wl',wl,'max_el',max_el,...
             'dt',dt,'g',grade);

%% Repeat STEPS 1-3 for a high resolution domain for High Res New York Part
min_el    = 30;               % minimum resolution in meters.
max_el    = 1e3;              % maximum resolution in meters.
max_el_ns = 240;              % maximum resolution nearshore.

coastline = 'PostSandyNCEI';
dem       = 'PostSandyNCEI.nc';

%polygon boubox
bbox2 = [-74.25 40.5; -73.75 40.55; -73.75 41; -74 41; -74.25 40.5];
gdat2 = geodata('shp',coastline,'dem',dem,'h0',min_el,'bbox',bbox2);

fh2 = edgefx('geodata',gdat2,'fs',R,'wl',wl,...
             'max_el',max_el,'max_el_ns',max_el_ns,...
             'dt',dt,'g',grade);

%% STEP 4: Pass your edgefx class object along with some meshing options
%% and build the mesh...
mshopts = meshgen('ef',{fh1 fh2},'bou',{gdat1 gdat2},...
                  'plot_on',1,'proj','lam');
```

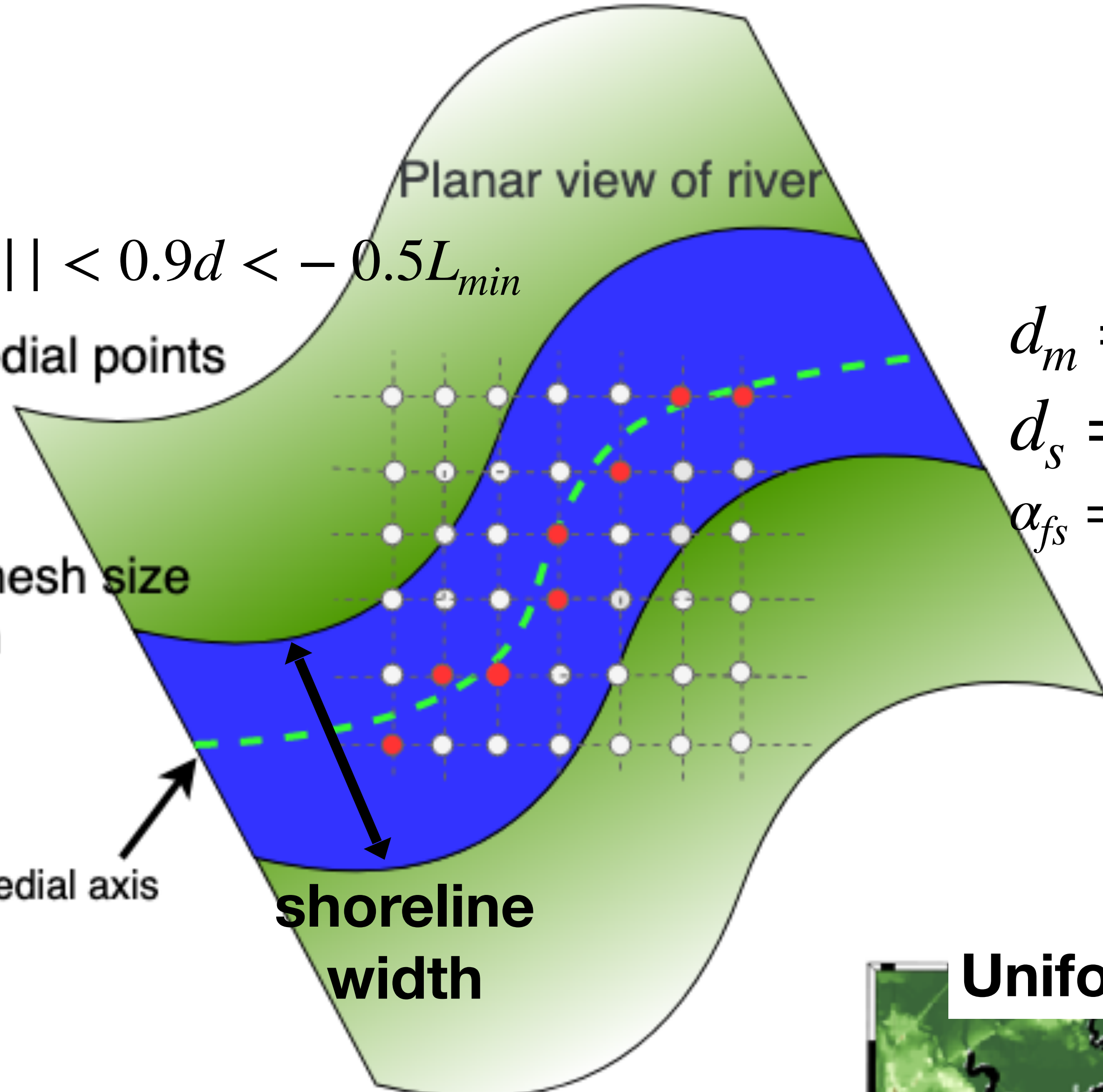


Multiscale meshing



Estimating shoreline width (feature size)

$p_M \rightarrow p: ||\nabla d|| < 0.9 d < -0.5 L_{min}$



$$h_{fs} = \frac{width}{\alpha_{fs}} = \quad h_{fs} = 2\frac{(d_m - d_s)}{\alpha_{fs}}$$

$d_m =$ nearest distance to medial points

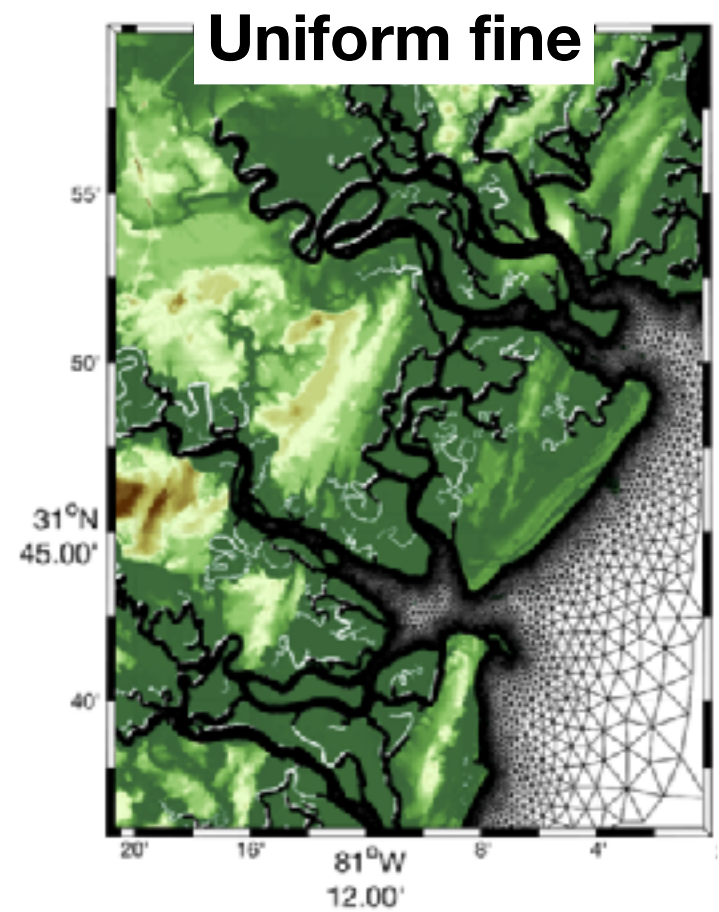
$d_s =$ **signed** nearest distance to shoreline points

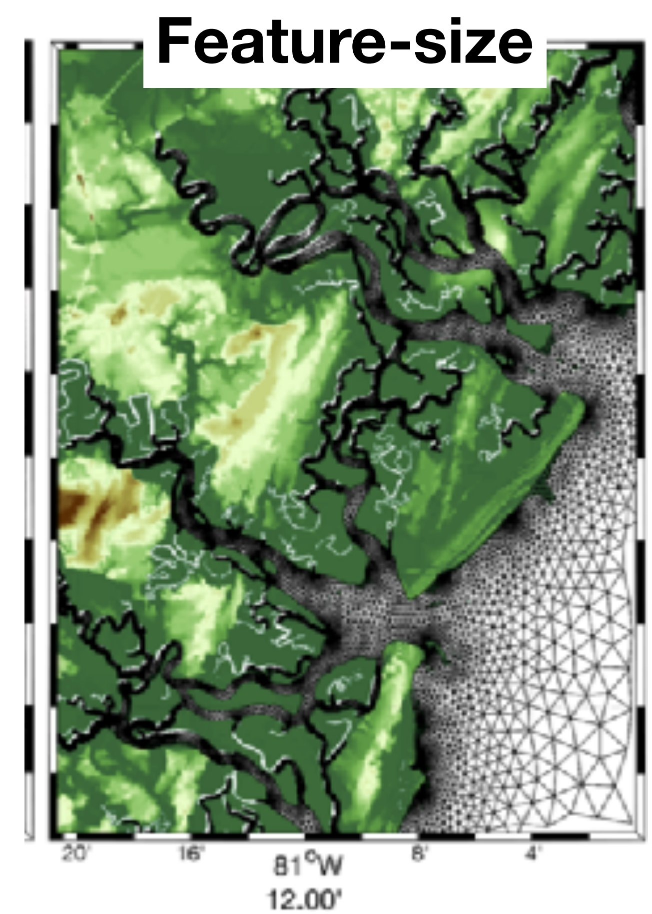
$\alpha_{fs} =$ number of elements per width**

$p_M =$ medial points

$L_{min} =$ **minimum element size**

$W =$ width of shoreline

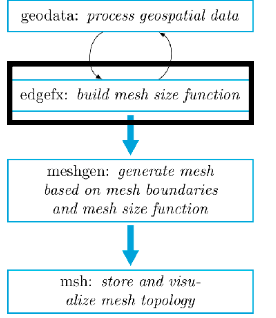




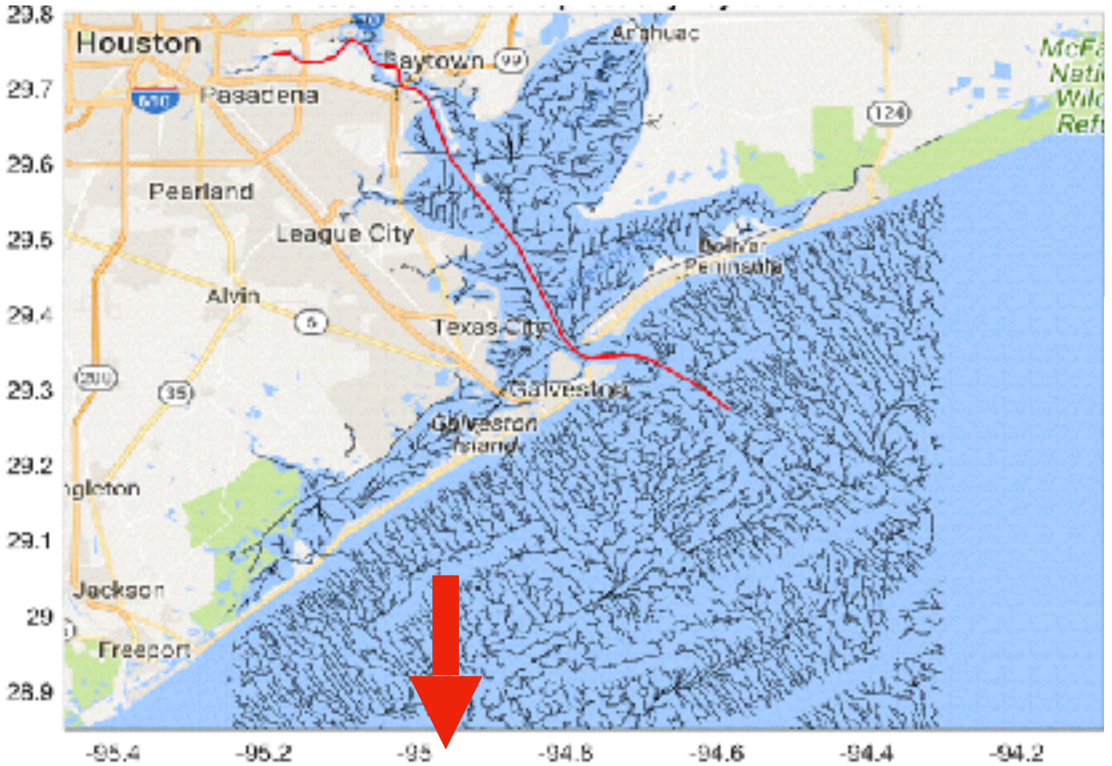
Improved upon work by:

Koko, Jonas, 2015. "[A Matlab mesh generator for the two-dimensional finite element method]()," [Applied Mathematics and Computation](), Elsevier, vol. 250(C), pages 650-664.

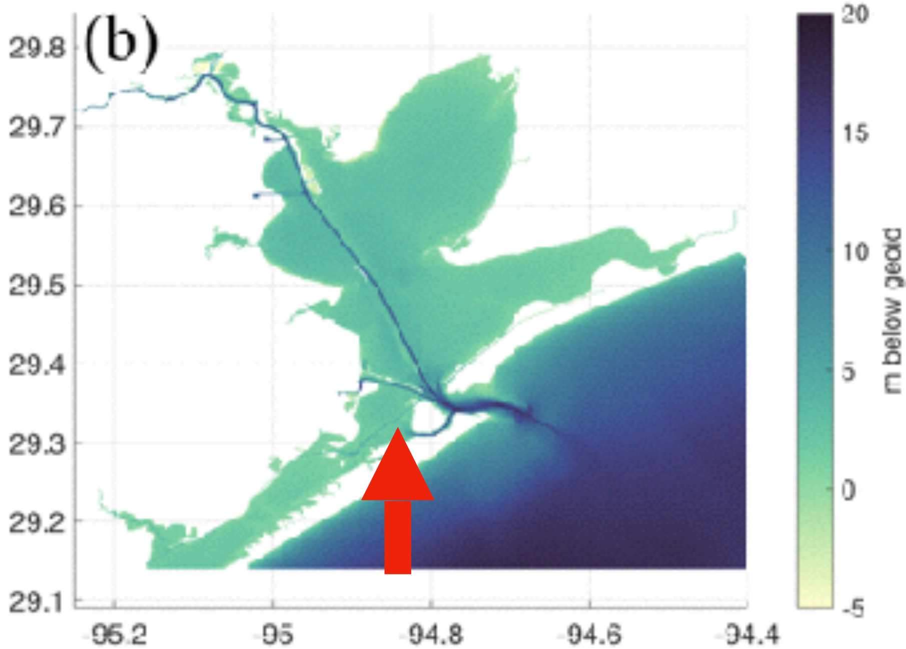
1. Recovery of "lost" medial points.
2. Removal of spurious medial points.
3. Improved efficiency of signed distance calculation.
4. Scale-aware feature size



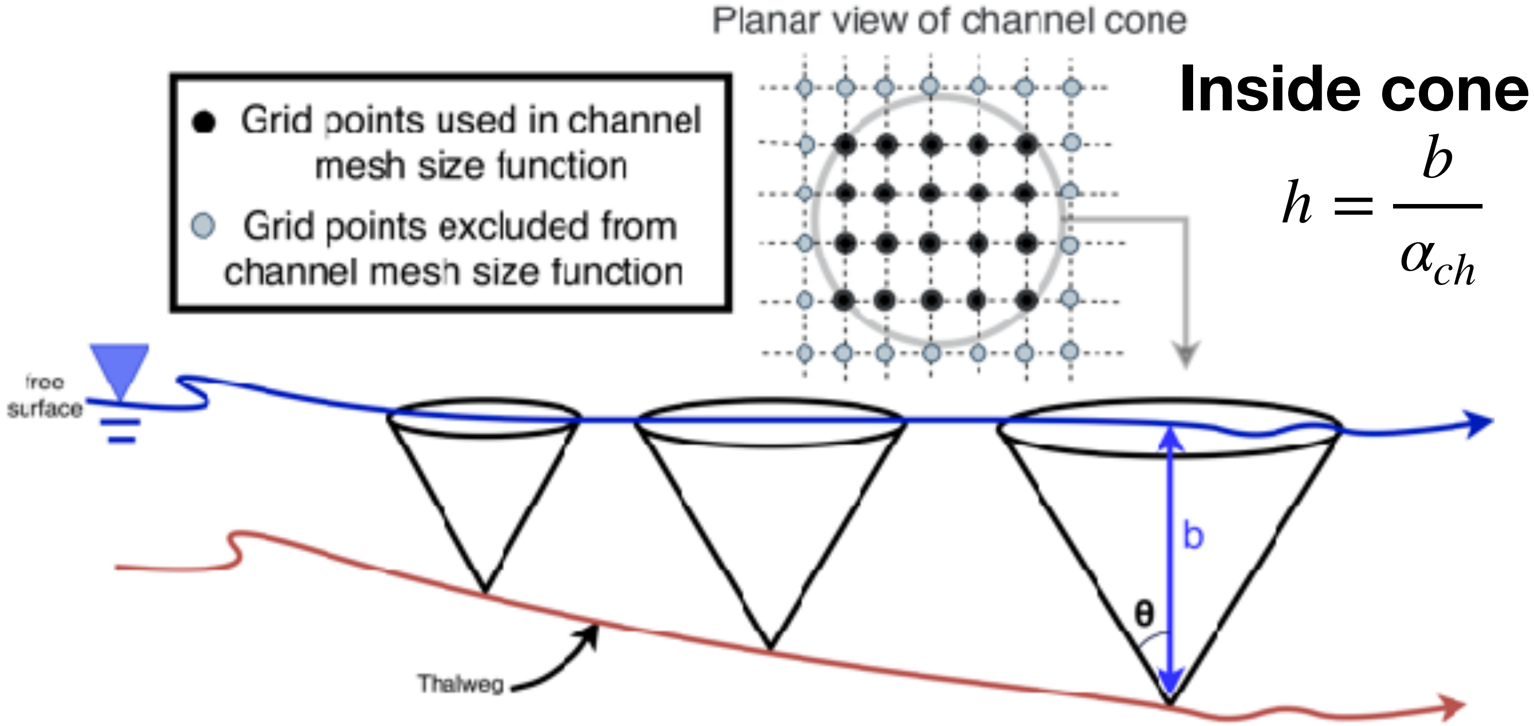
Input: stream network from DEM



More accurate interpolated seabed extrema



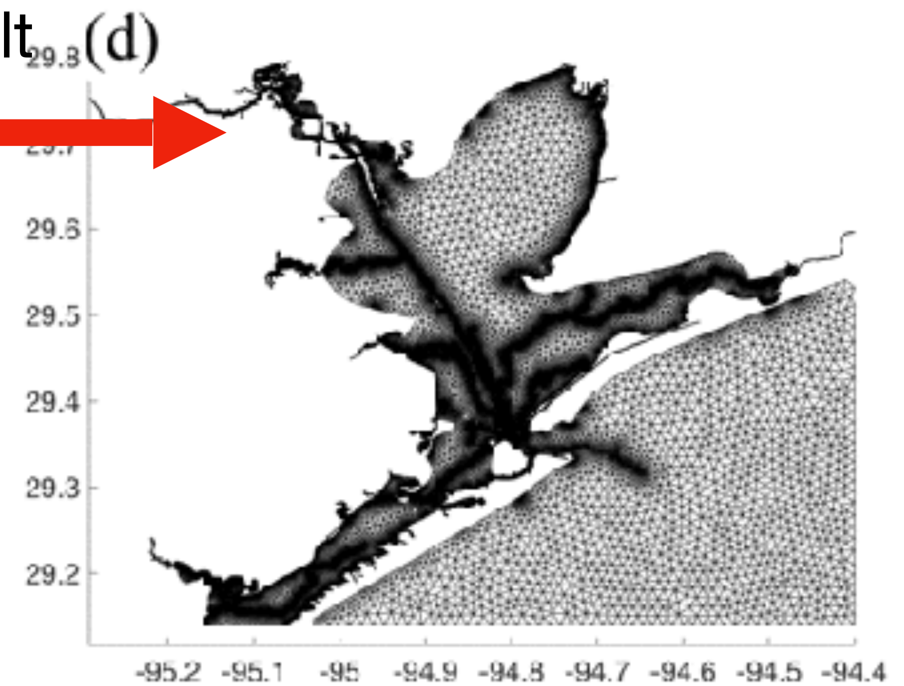
Mechanism of resolution distribution



Inside cone

$$h = \frac{b}{\alpha_{ch}}$$

Result



Mesh size gradation

*Size transitions are bounded above by a limit **g**.*

Implemented a method to gradient limit mesh size functions based on the work of:

P.-O. Persson, 2006. Mesh size functions for implicit geometries and PDE-based gradient limiting. Engineering with Computers.

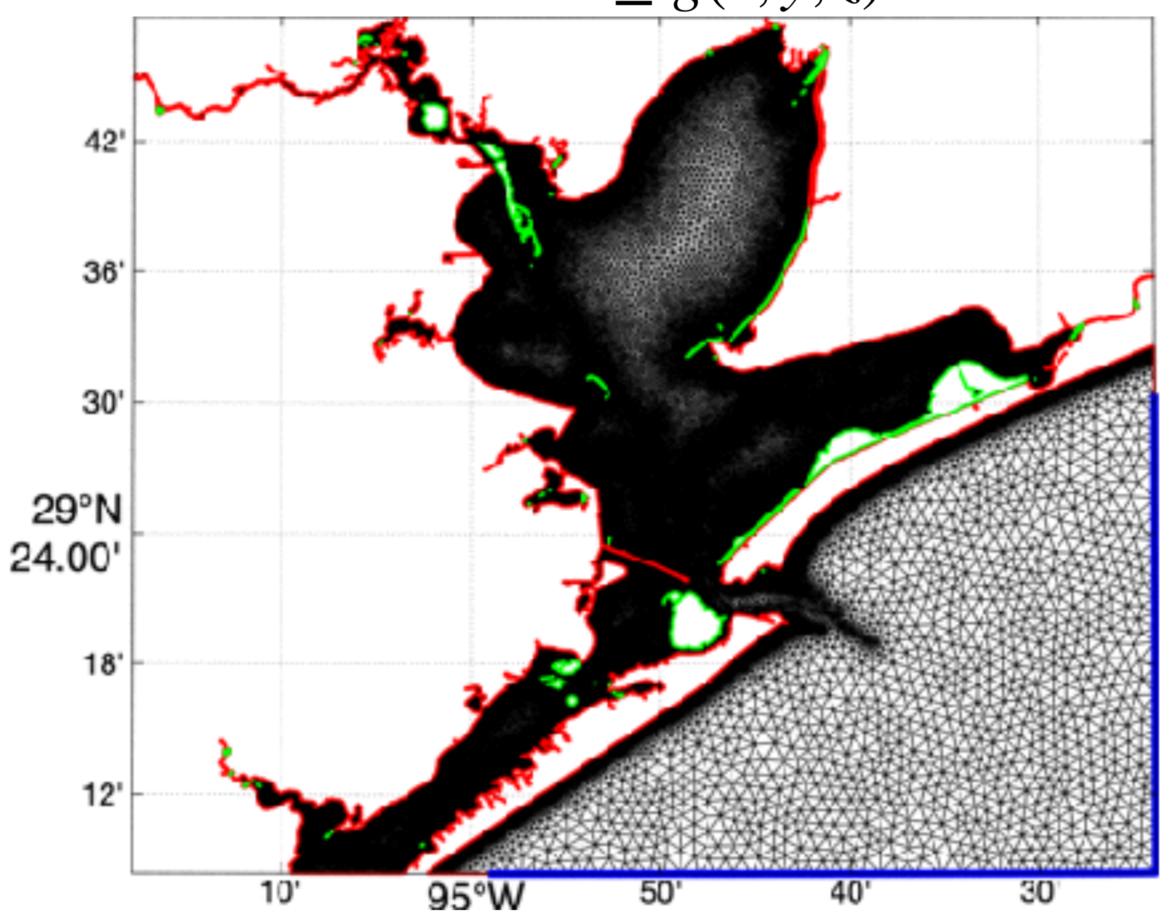
Solve the following on a Cartesian grid

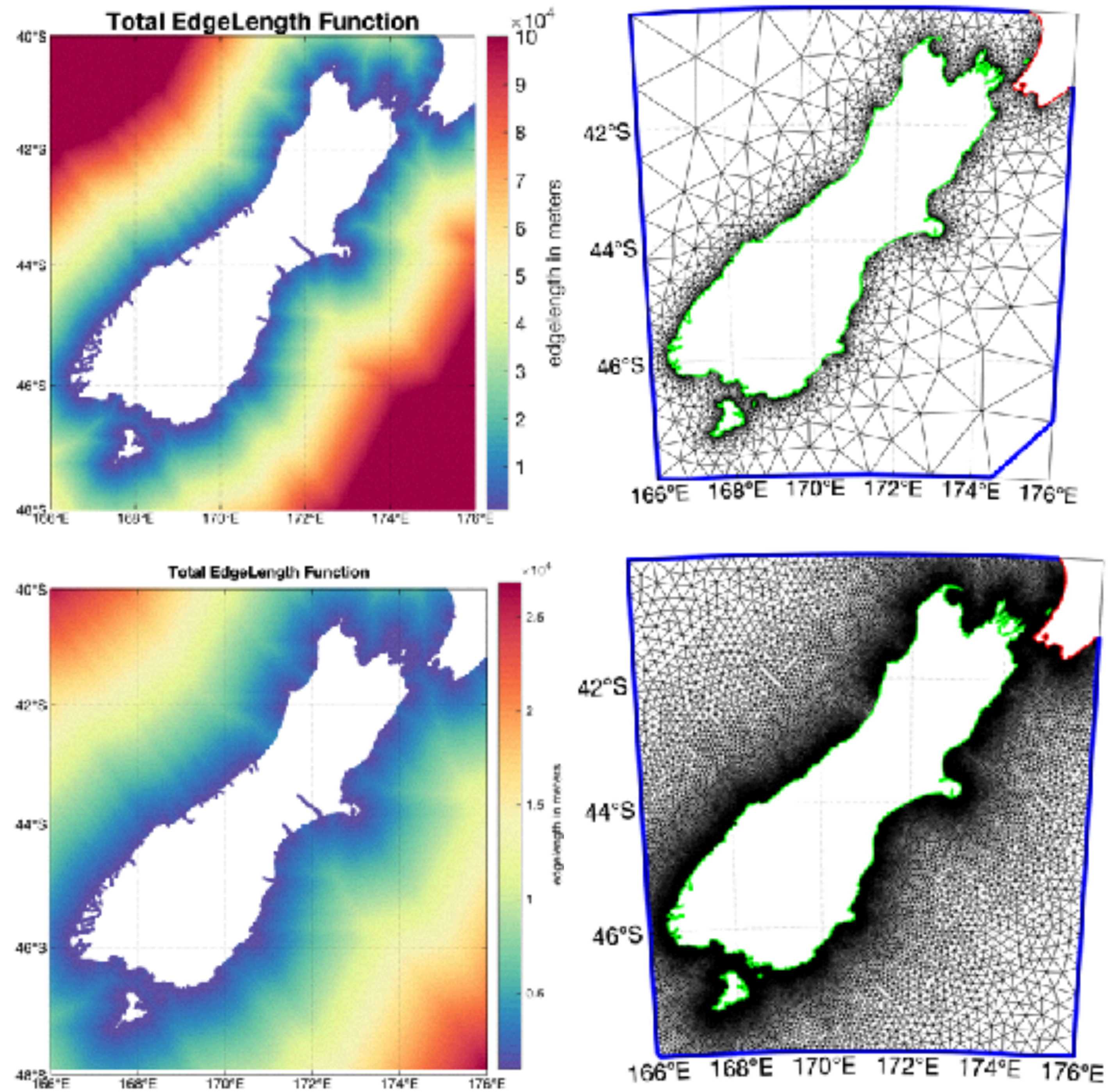
$$\frac{\partial h}{\partial t} + |\nabla h| = min(|\nabla h|, g(x,y,z))$$

Note when $\nabla h \leq g$ then $\frac{\partial h}{\partial t} = 0$

h edgelength of triangle

$\nabla h \leq g(x,y,z)$





Topographic-lengthscale

$$h = \frac{2\pi}{x} \frac{b}{|\nabla b|}$$

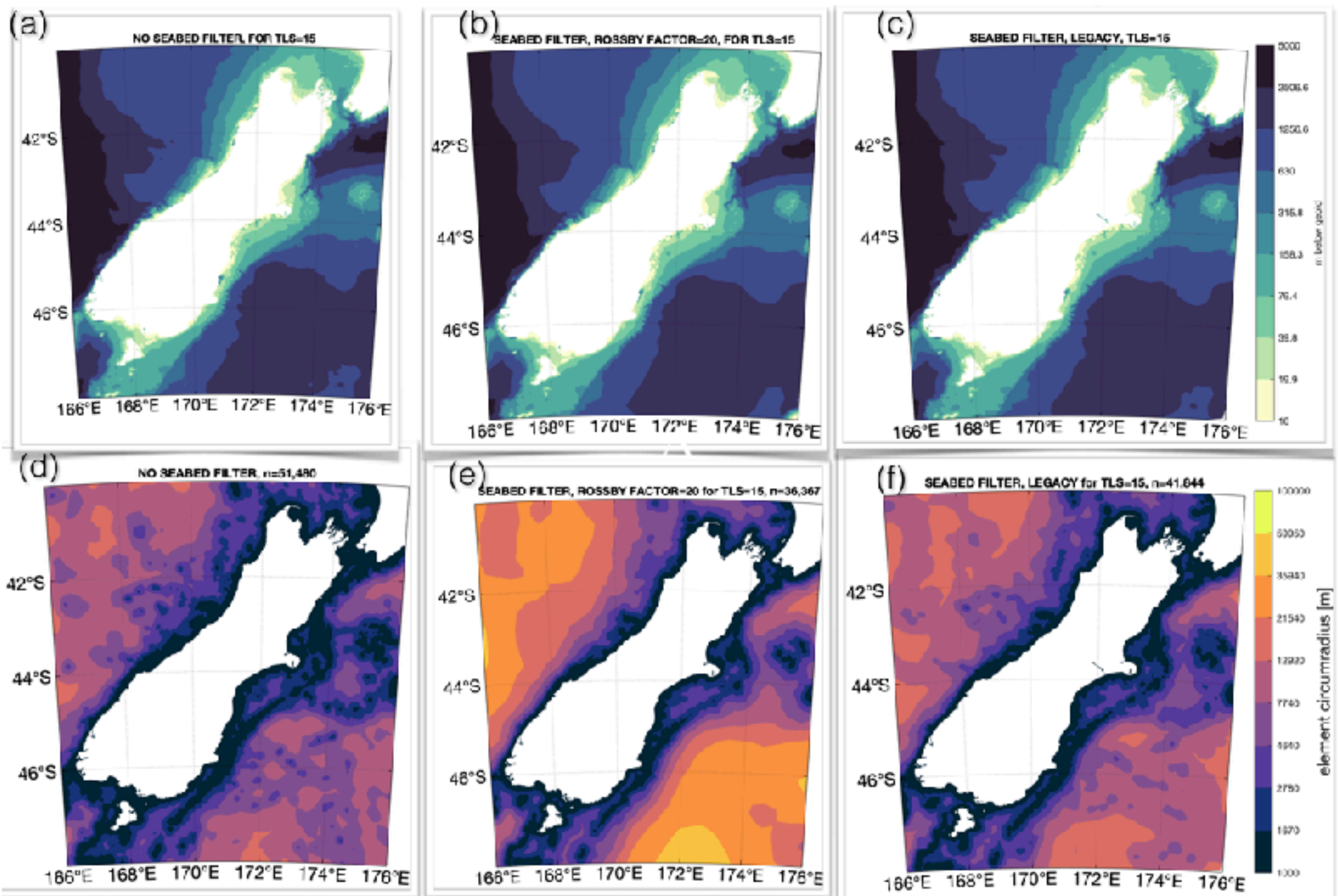
How do you filter b?

$$L_R = \frac{\sqrt{gd}}{f}$$

No filter

Rossby factor w/ Gaussian filter

Rossby factor w/ ∇^2

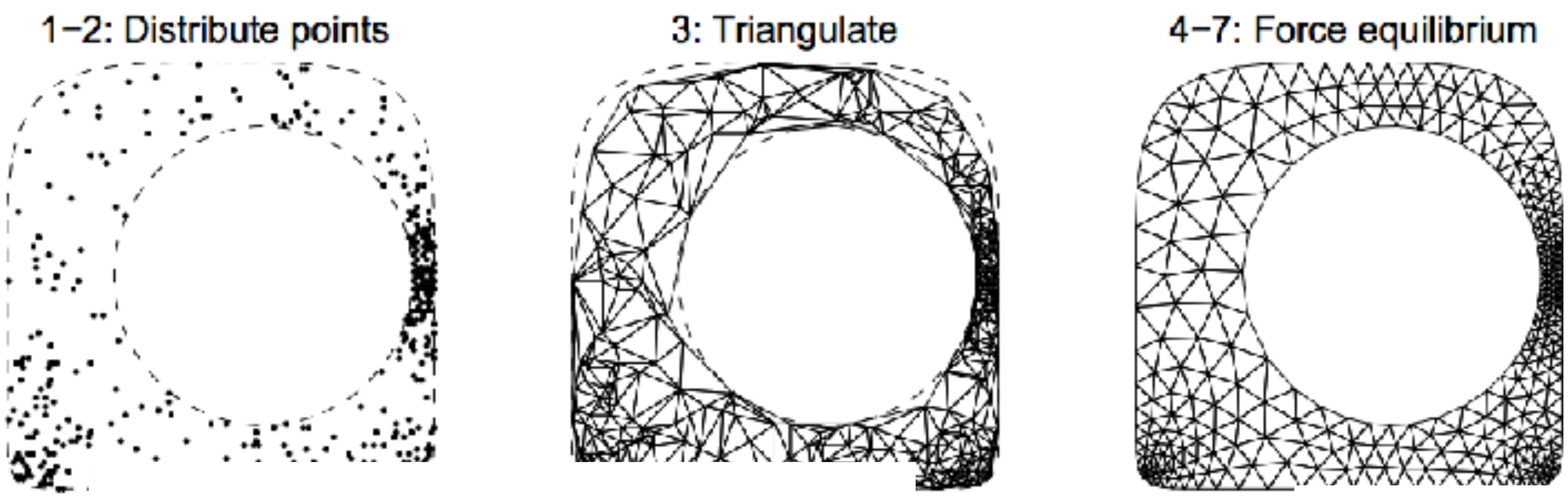


Meshgen: generation via modified *DistMesh2D* algorithm

- P.-O. Persson, G. Strang, **A Simple Mesh Generator in MATLAB**.

  SIAM Review, Volume 46 (2), pp. 329-345, June 2004 (PDF)

$p(0) = p_0$ → ***Delaunay triangulation*** ***Move points based on F*** → $F(p) = 0$



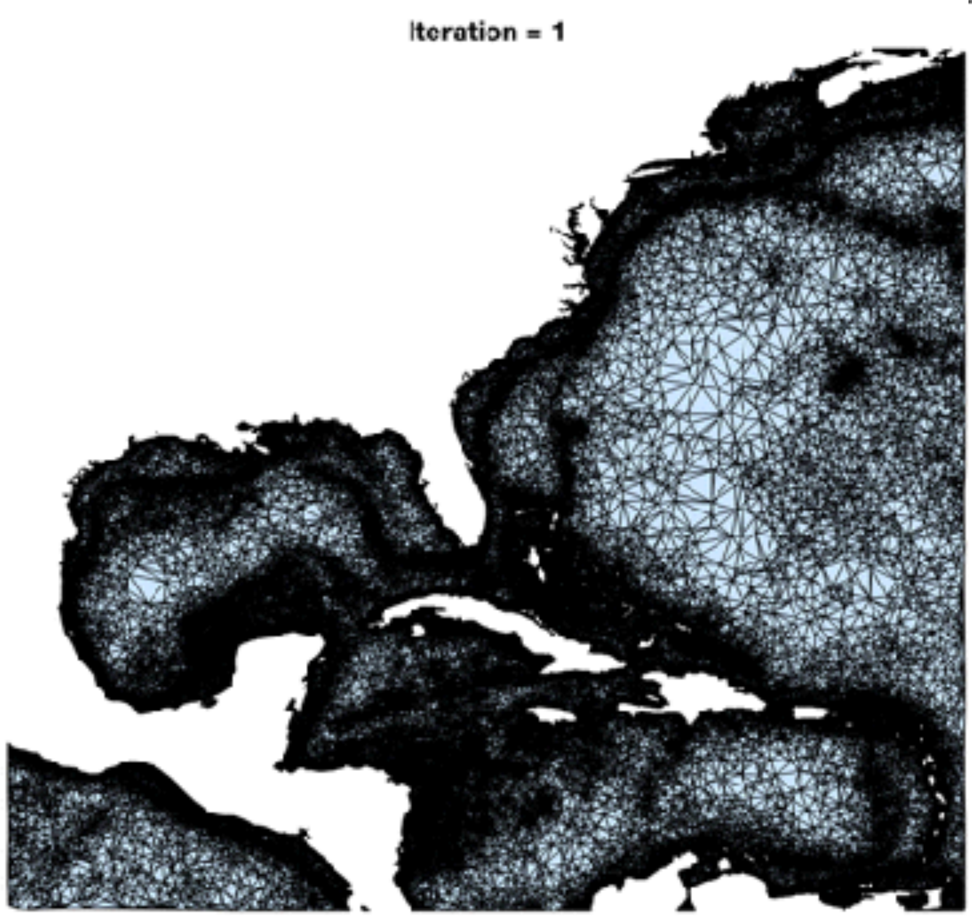
$\mathbf{p} = [xy]$

$\mathbf{F}(\mathbf{p}) = [\mathbf{F}_x(\mathbf{p})\mathbf{F}_y(\mathbf{p})]$

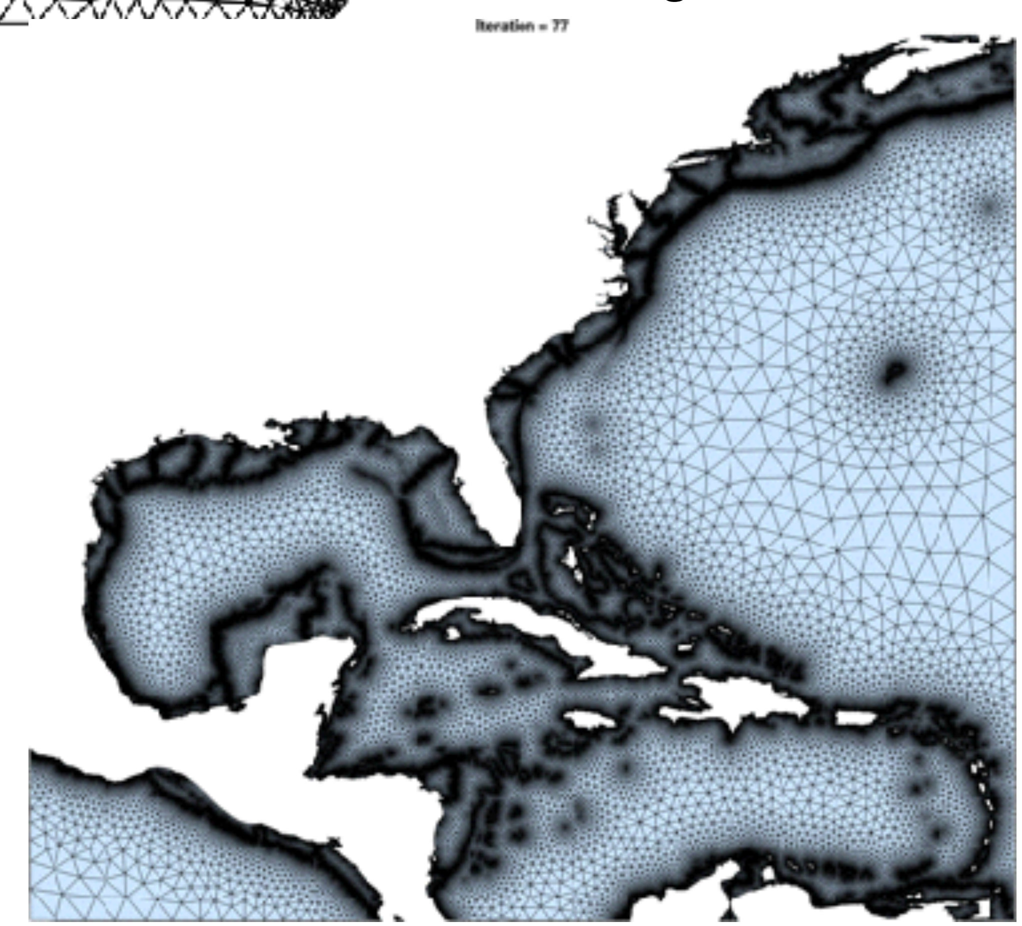
$$\frac{dp}{dt} = \mathbf{F}(\mathbf{p}), t \geq 0$$

$$\mathbf{p_{n+1}} = \mathbf{p_n} + \Delta t\mathbf{F}(\mathbf{p_n})$$

****Points that exit are reprojected back into meshing domain**



$\mathcal{O}(minutes)$ →



User passes the edgefx and geodata class instances

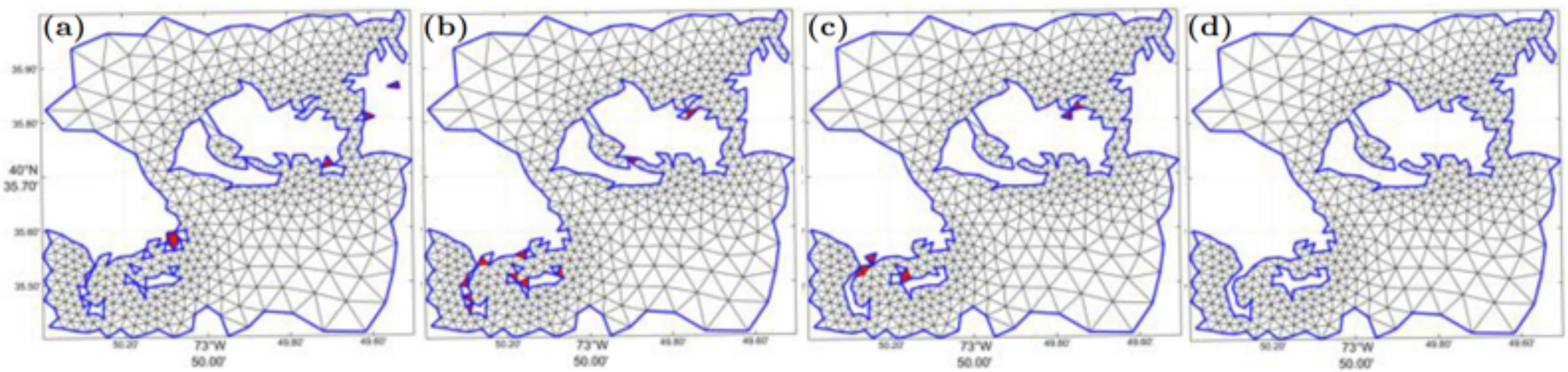
```
%% STEP 4: Pass your edgefx class object along with some meshing options and
% build the mesh...
mshopts = meshgen('ef',fh,'bou',gdat,'plot_on',1,'proj','lambert');
% now build the mesh with your options and the edge function.
mshopts = mshopts.build;
%% STEP 5: Plot it and write a triangulation fort.14 compliant file to disk.
m = mshopts.grd;
```

Boundary simplification

Cleaning algorithm leads to a valid mesh boundary for our finite element methods.

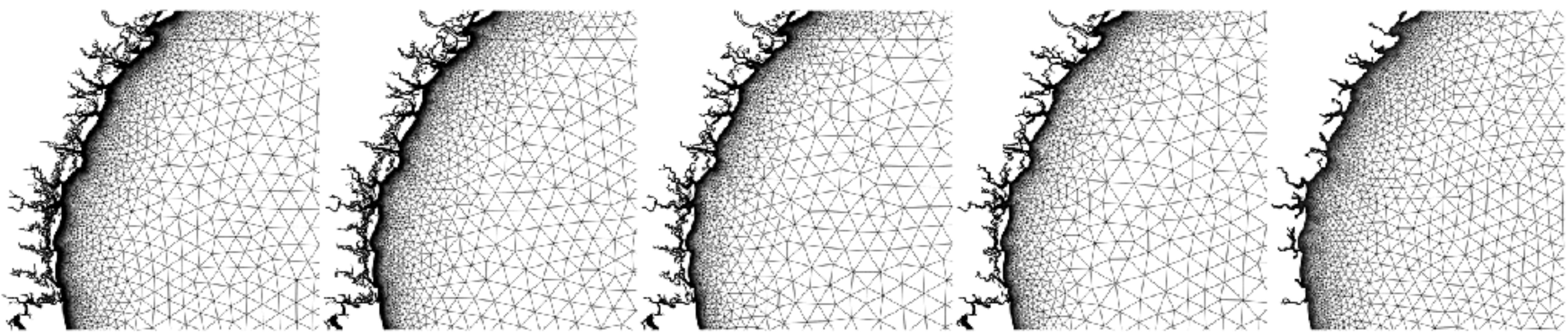
Repeat until no more geometric violations

Delete exterior disjoint → Delete interior disjoint → Delete exterior disjoint → Cleaned



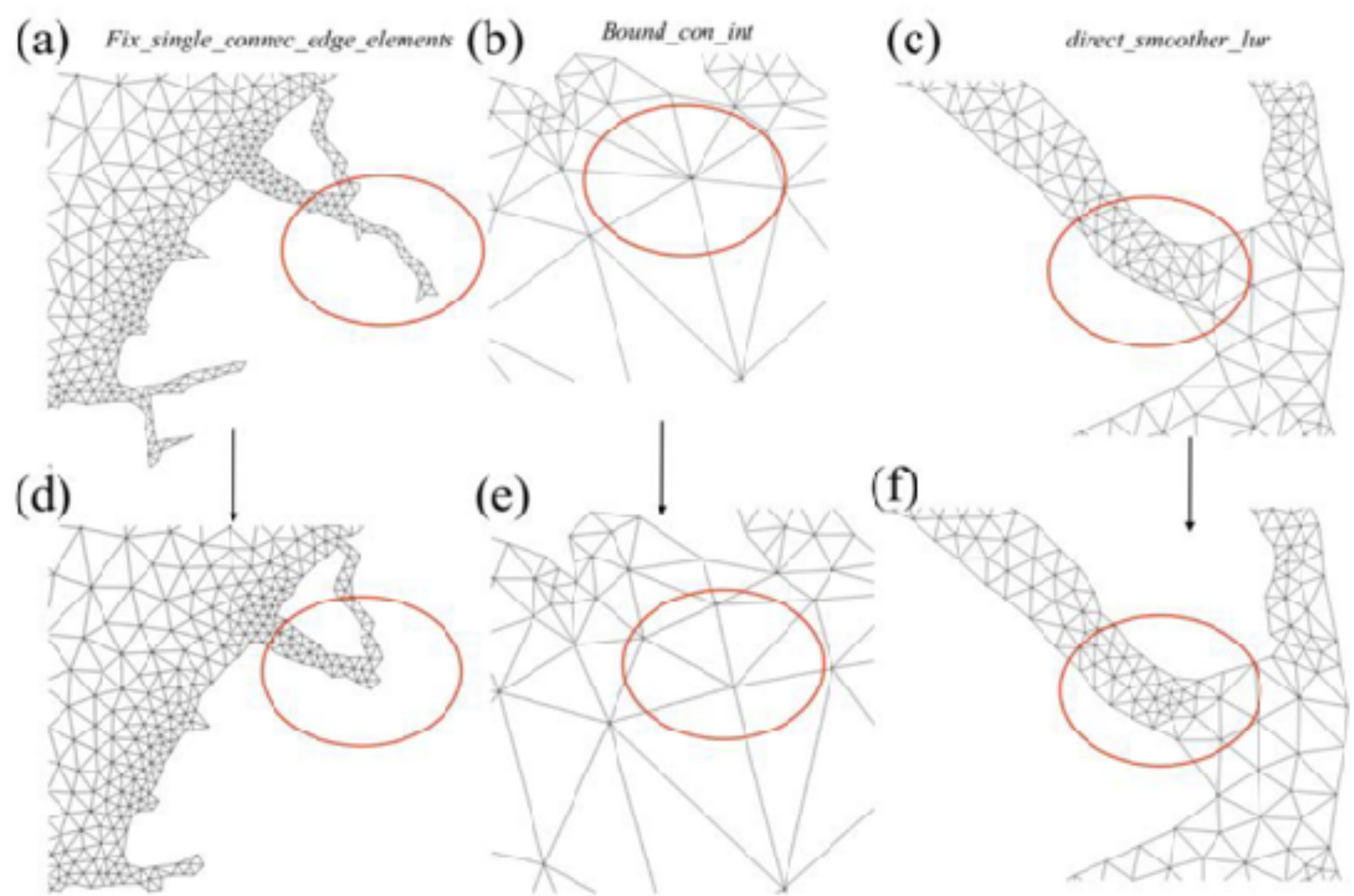
Number of boundary vertices = number of boundary segments

Finer shoreline resolution → Coarser shoreline resolution

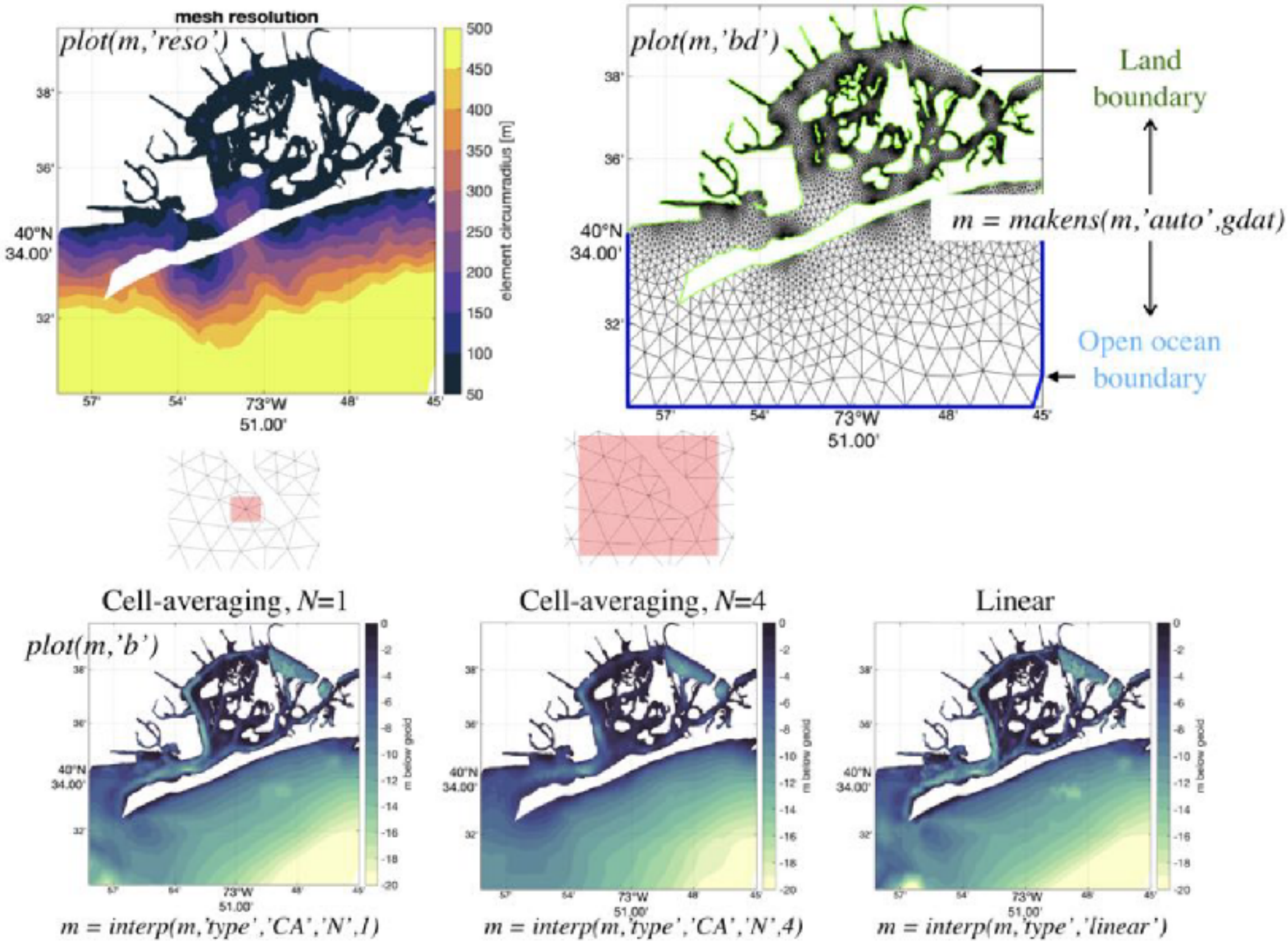


Mesh improvement/boundary simplification

```
>> help m.clean
--- help for msh/clean ---

  [obj,qual] = clean(obj,varargin)
  obj - msh object
  varargin - optional base cleaning type followed by optional
  name-value pairs as listed below:

  base cleaning types: 'medium' (or 'default'), 'passive', 'aggressive'

  optional name-value pairs
  'db'  - boundary element cutoff quality (0 - 1)
  'ds'  - perform direct smoother? (0 or 1)
  'con' - upper bound on connectivity (6-19)
  'djc' - dj_cutoff (0 - 1 [area portion] or > 1 [km^2])
  'sc_maxit' - max iterations for deletion of singly connected
          elements ( >= 0, if set to 0 operation not performed)
  'mqa' - allowable minimum element quality (0 - 1); setting
          this too value high may prevent convergence
  'nscreen' - print info to screen? (default = 1)
  'pfix' - fixed points to keep (default empty)
  'proj' -to project or not (default = 1)

>> |
```



Msh class



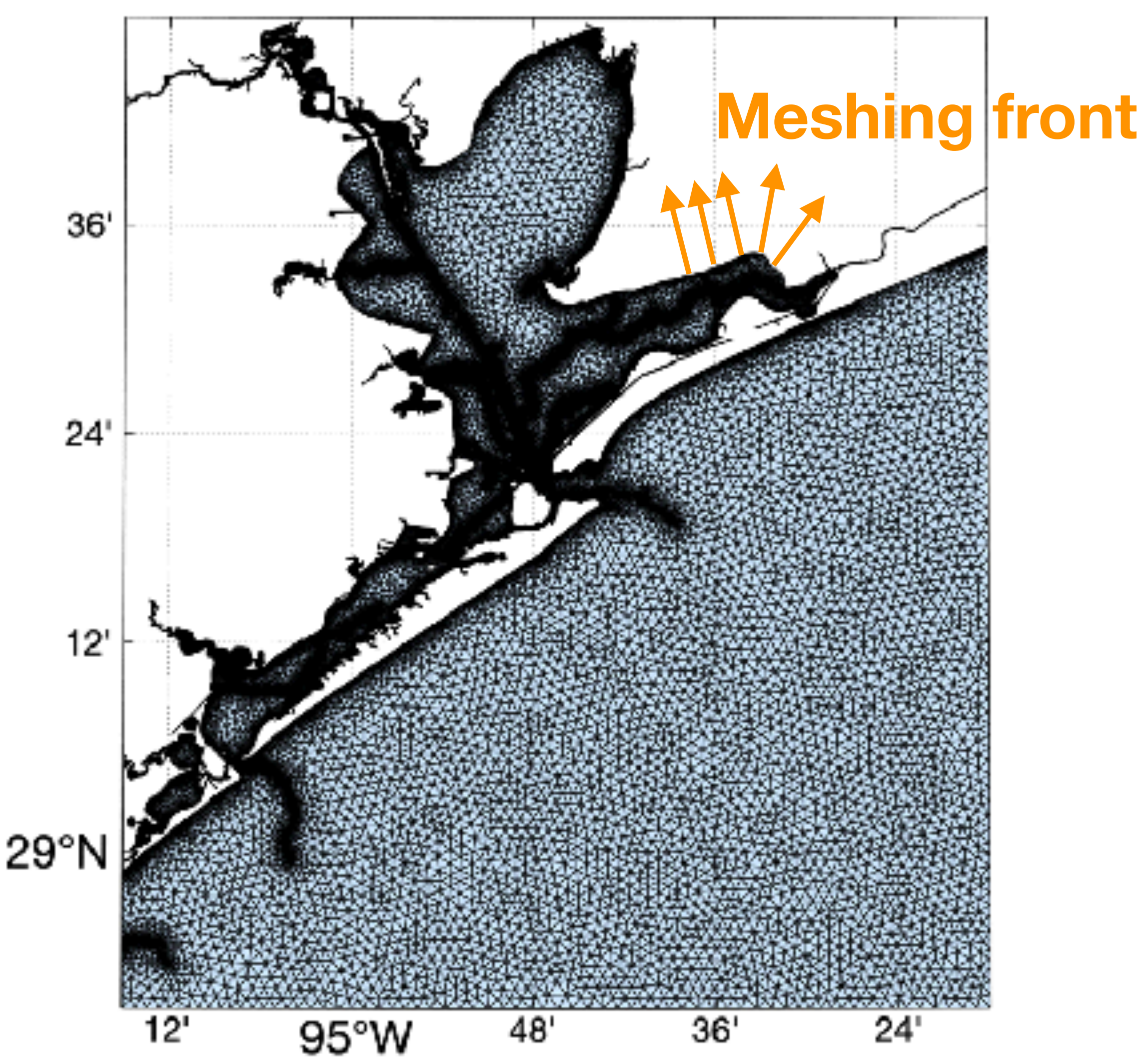
mesh resolution
plot(m,'reso')
element circumradius [m]
plot(m,'bd')
Land boundary
m = makens(m,'auto',gdat)
Open ocean boundary
40°N 34.00'
73°W 51.00'
Cell-averaging, N=1
Cell-averaging, N=4
Linear
plot(m,'b')
m below geoid
m = interp(m,'type','CA','N',1)
m = interp(m,'type','CA','N',4)
m = interp(m,'type','linear')

Floodplain meshing

Three strategies (only two shown)

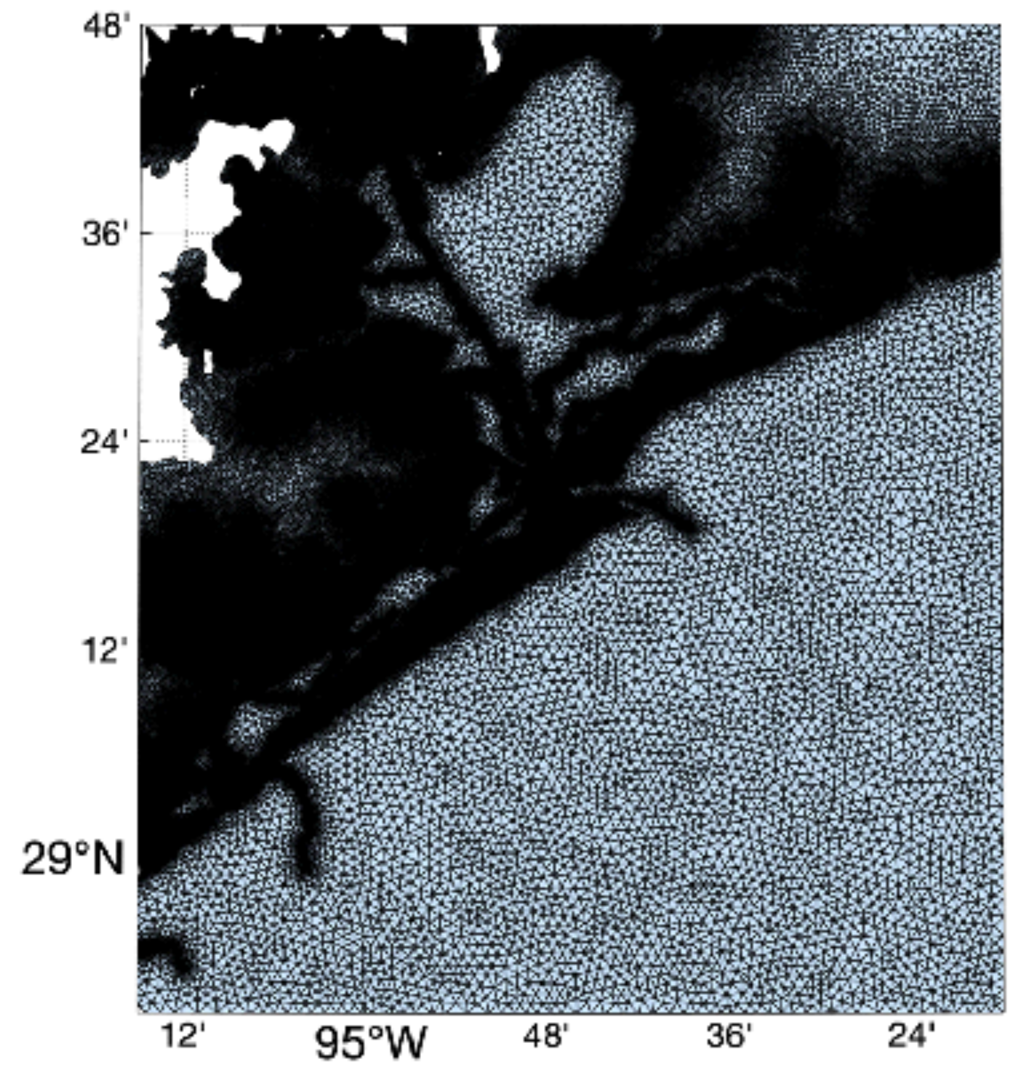
Del. refine (using Mesh2D)

1. Build oceanside of domain
2. Build floodplain mesh propagating meshing front from oceanside boundary



Internal point and edge constraints

1. Build oceanside of domain
2. Build up to floodplain contour while constraining boundary of oceanside mesh

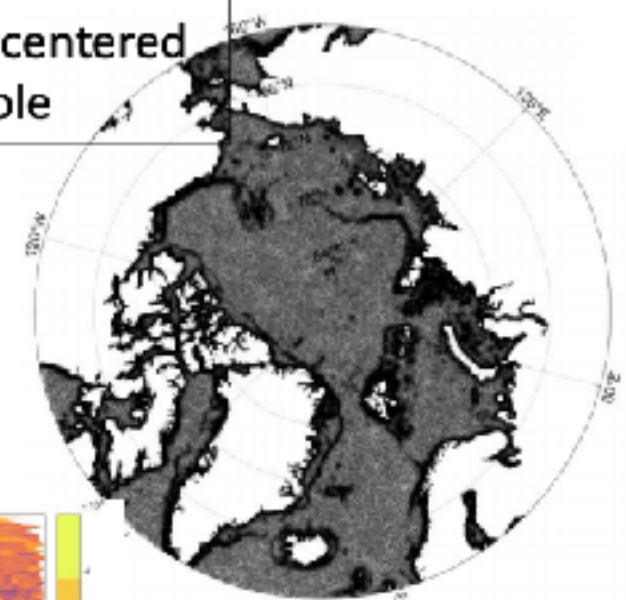


Global meshing

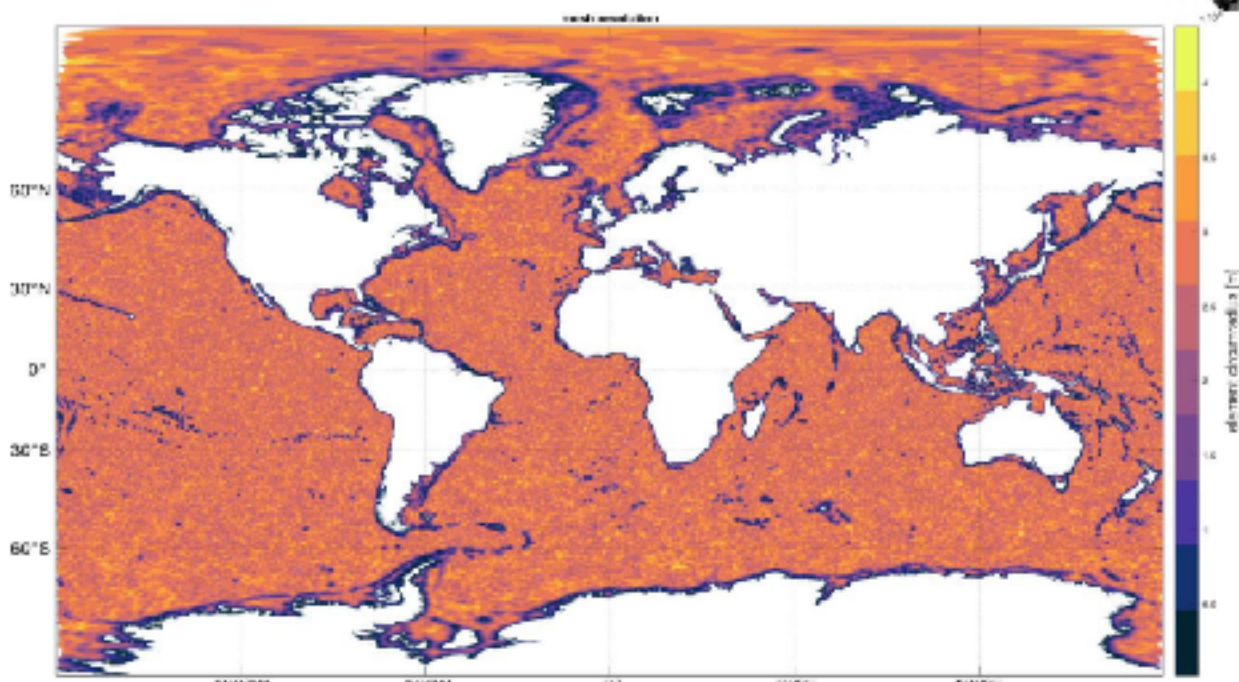
Meshes can built in a stereographic projection.
Boundary is periodic by construction.
No tidal BCs necessary

Note:
We have been able to use around **2 min time step** on this mesh. Equivalent to CFL of 13.
5-day forecast on 48 CPUs takes 10 minutes

Stereographic projection centered at North Pole

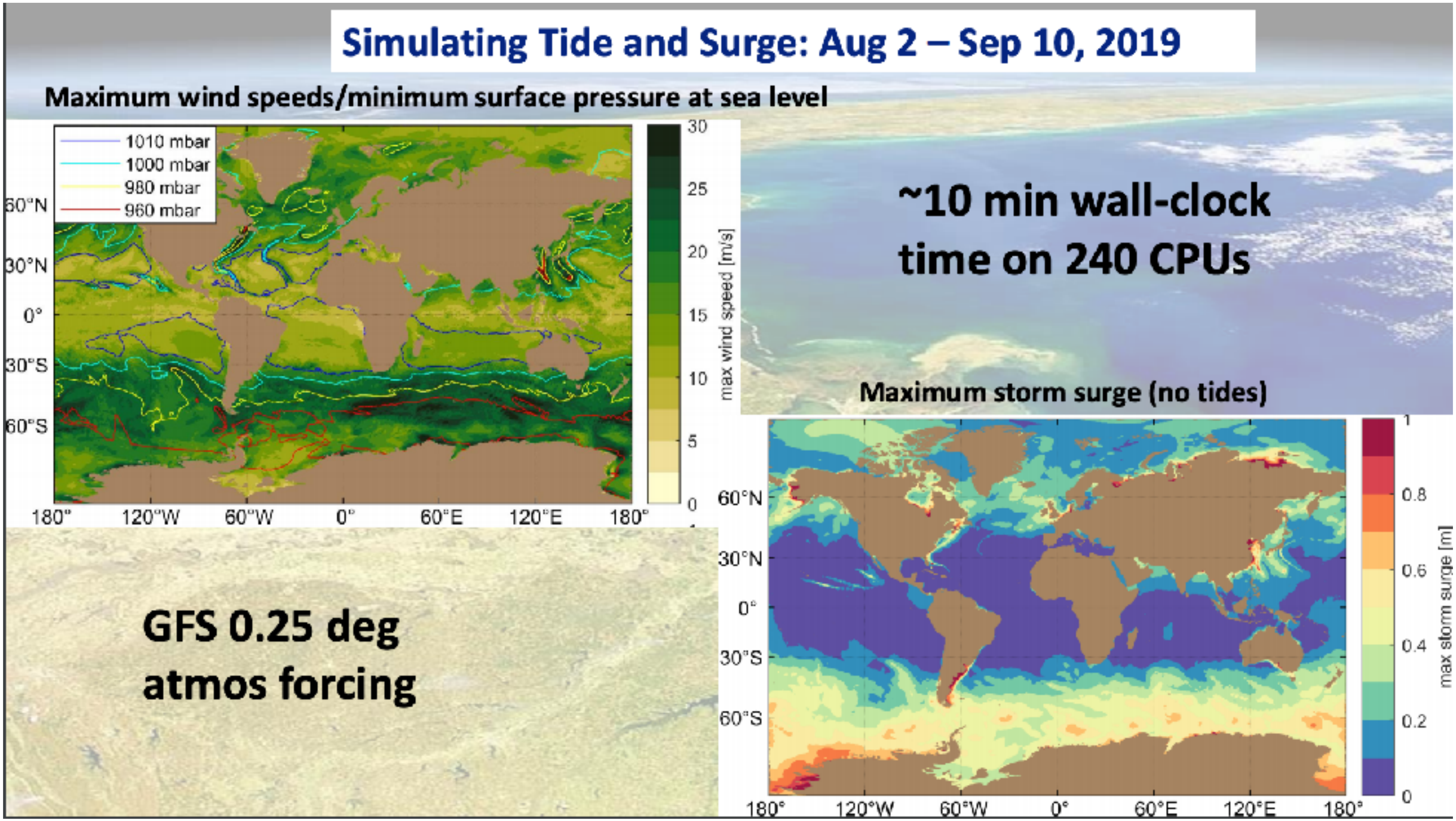


Nominal element sizes range ~2 km to 25 km

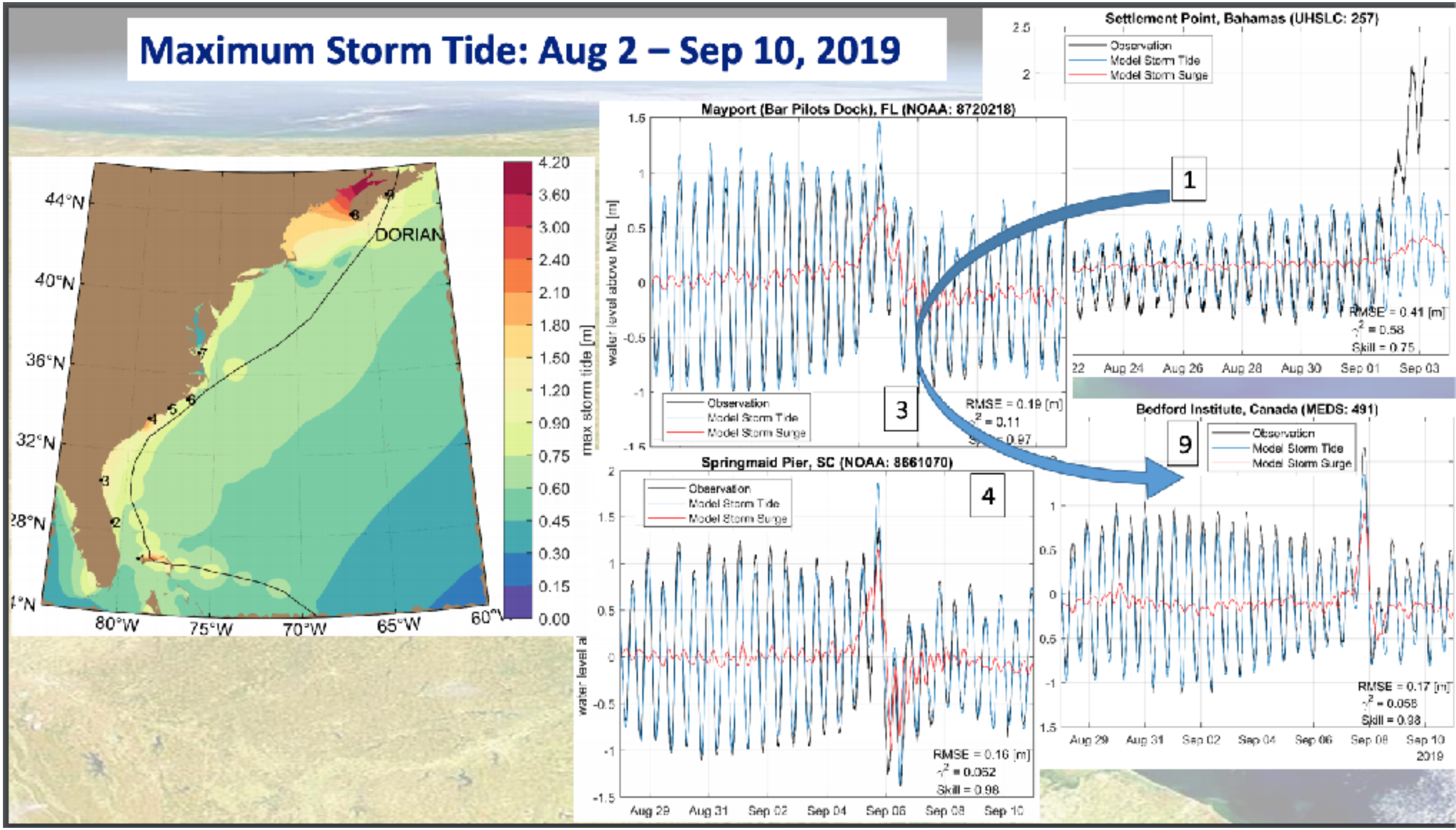


https://github.com/CHLNDDEV/OceanMesh2D/

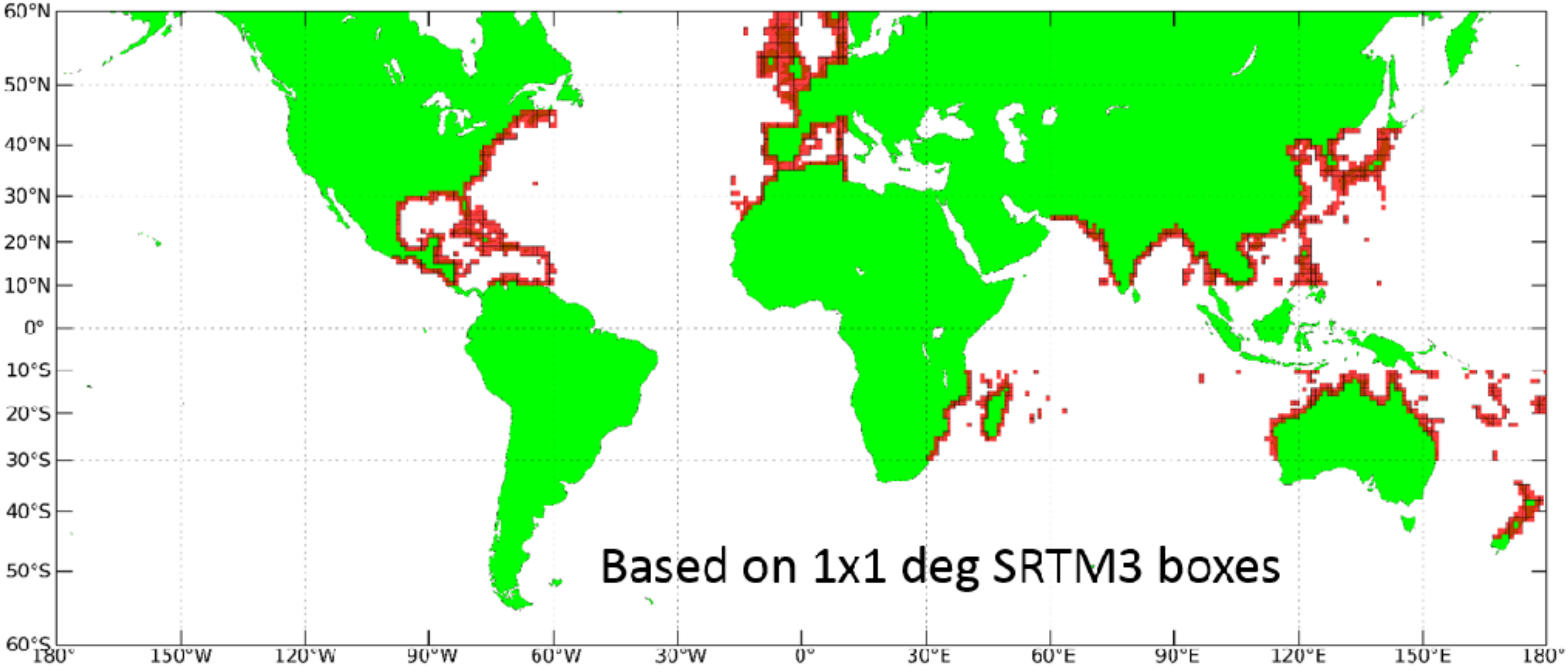
Global storm tides



Global storm tides



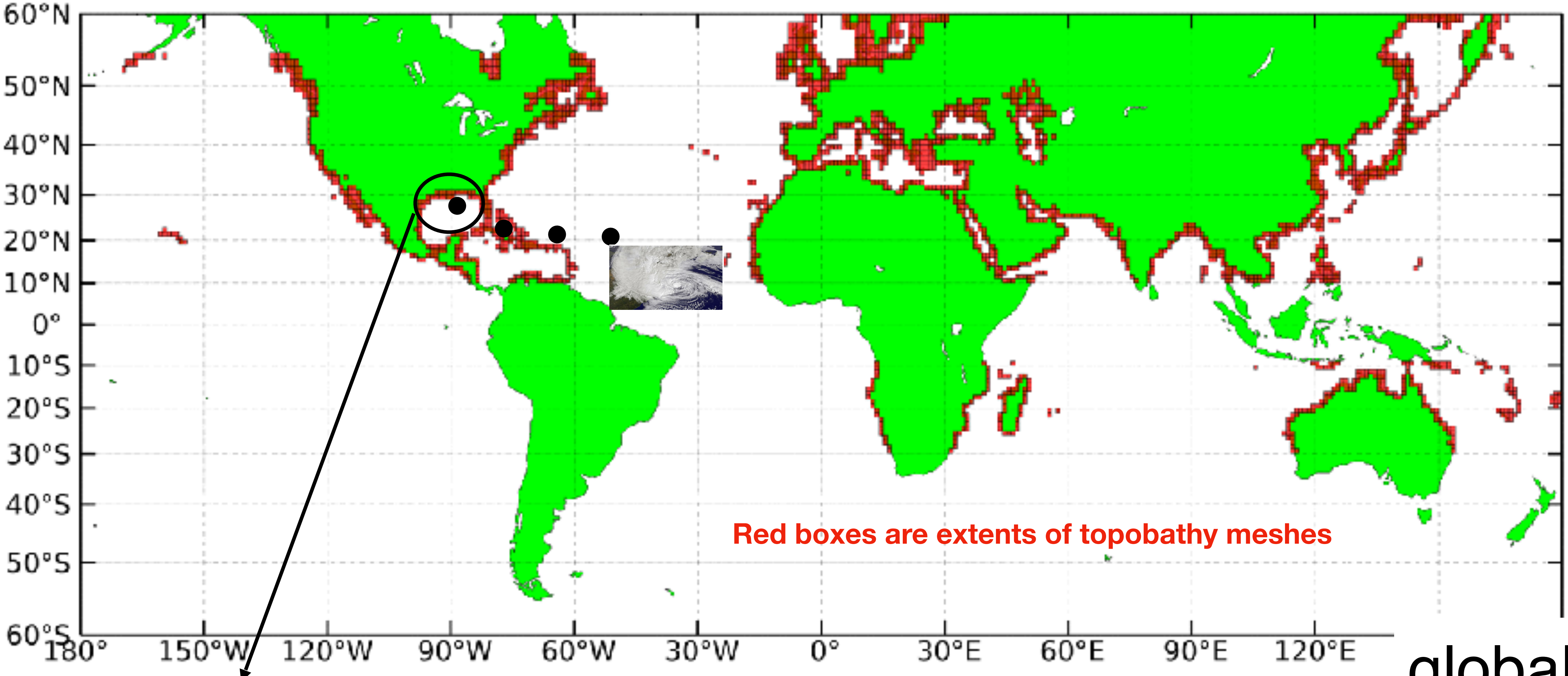
Automated global coastal water level modeling



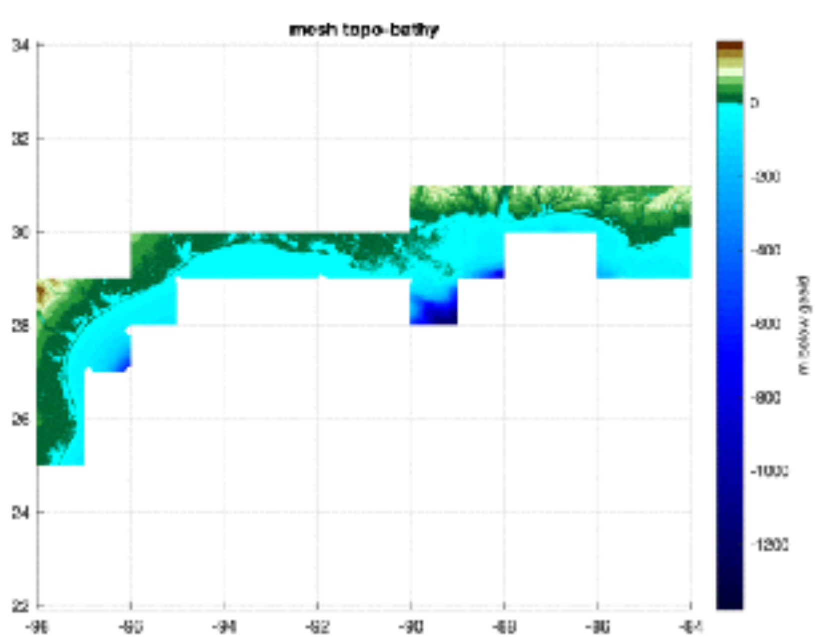
https://github.com/WPringle/GLOCOFFS

Automated global coastal water level modeling

Pre-assembled tiled meshes merged together "on-the-fly"

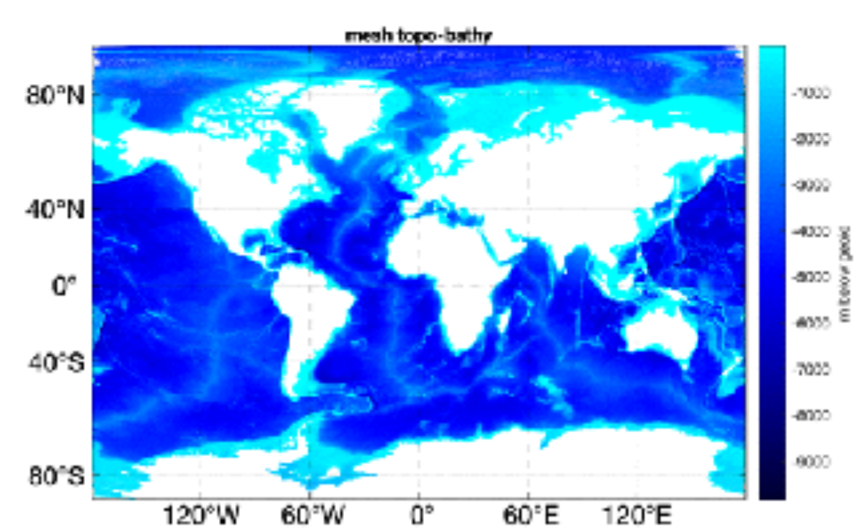


local hi-res mesh



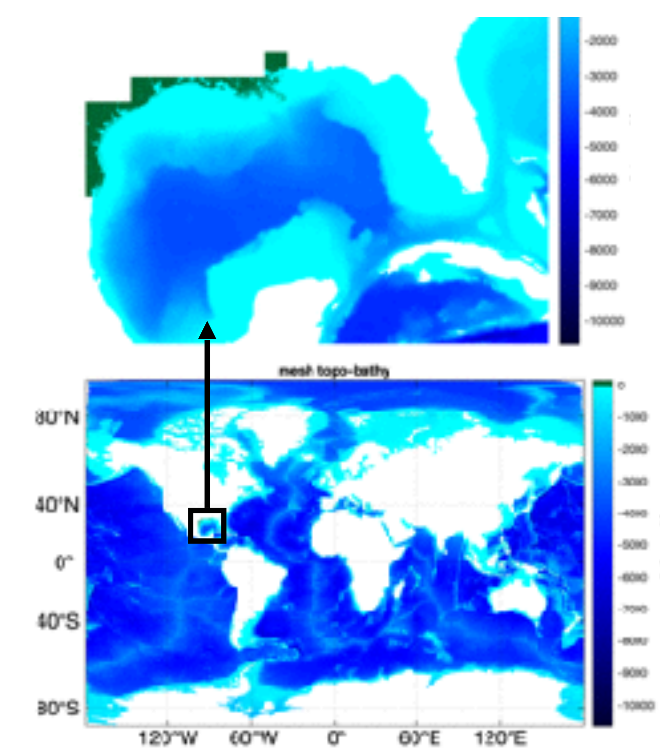
plus

global mesh

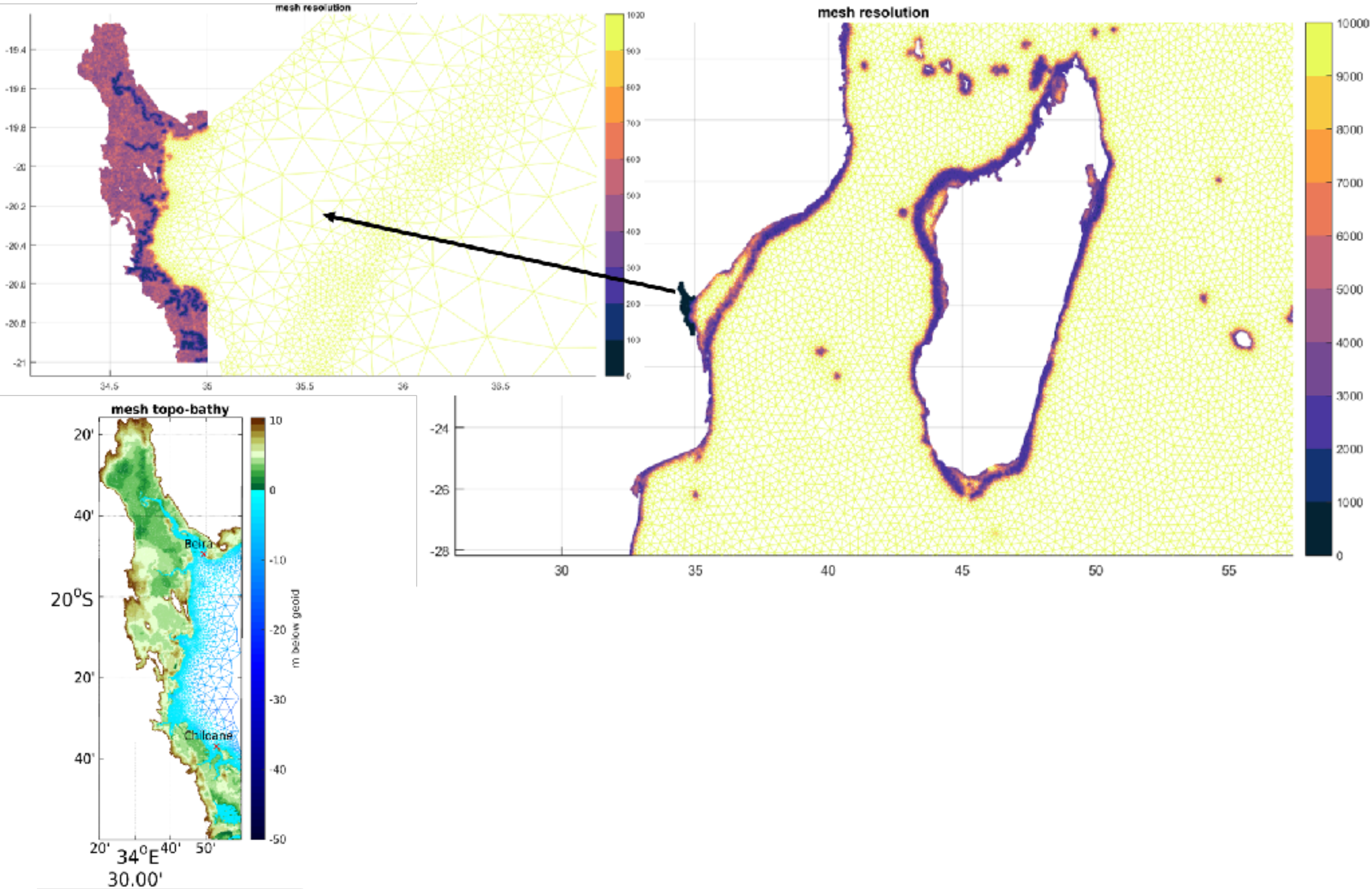


=

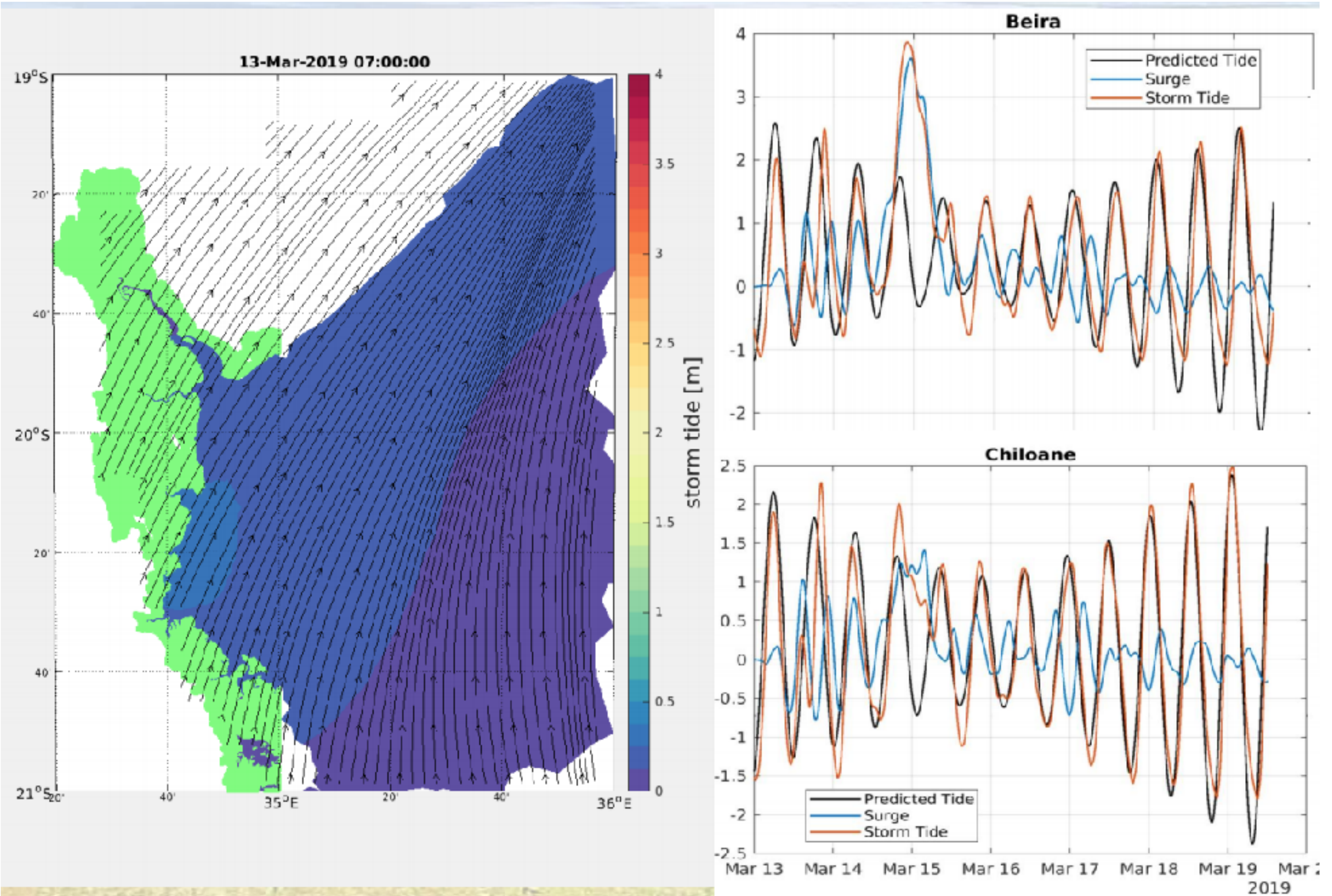
global + local



Automated global coastal water level modeling
Tropical Cyclone Idai – Mid-March 2019

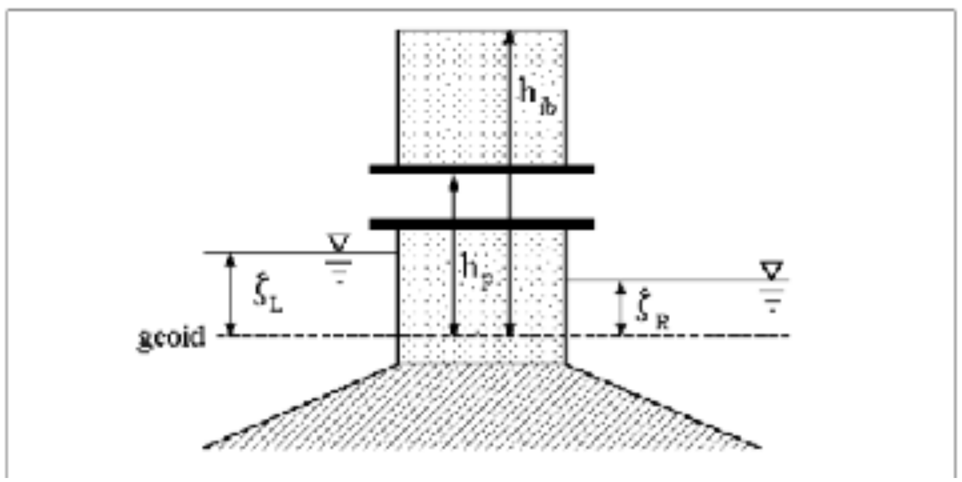


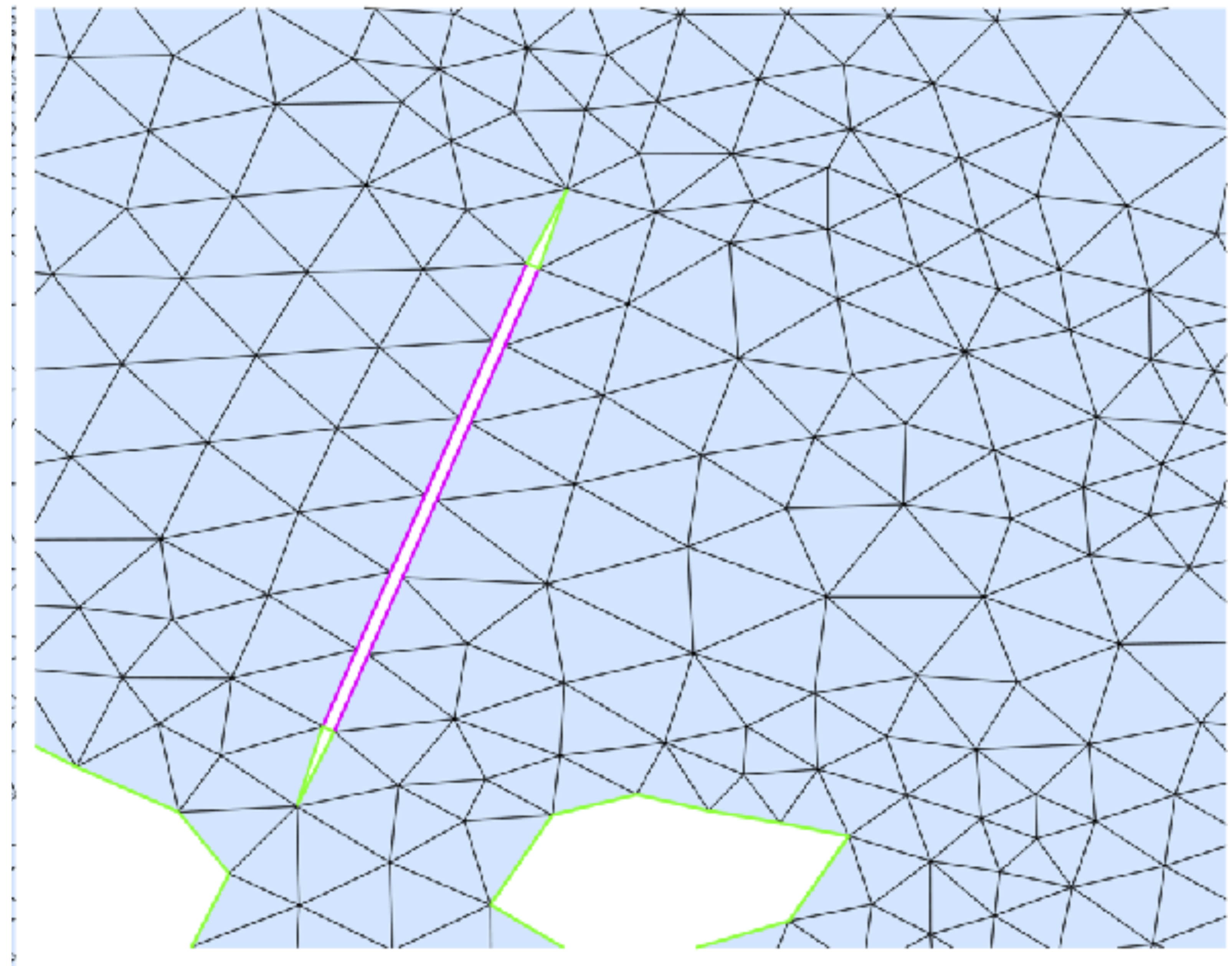
Tropical Cyclone Idai – Mid-March 2019



Meshing Barriers

Barriers are represented as weirs.

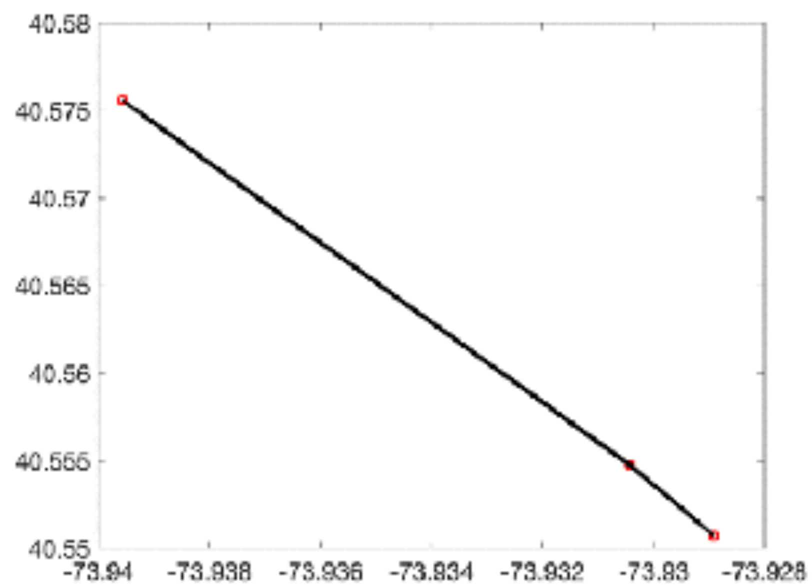


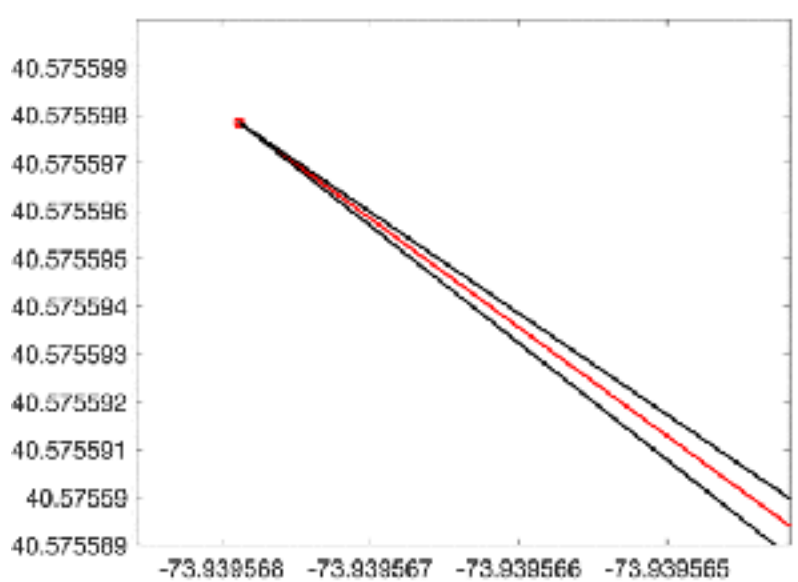


Meshing barriers

Crestline of barrier is passed

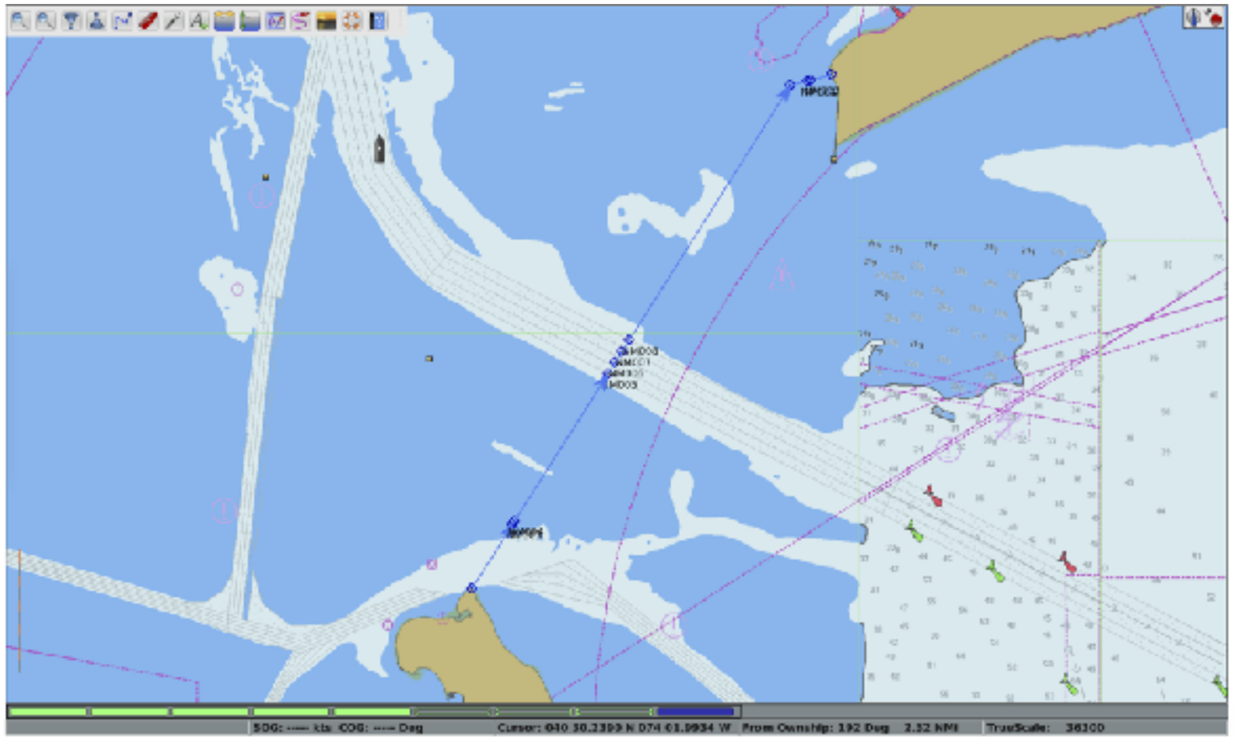
```
struct with fields:

          X: [3×1 double]
          Y: [3×1 double]
      width: 30
    min_ele: 100
crestheight: 4.3000
```

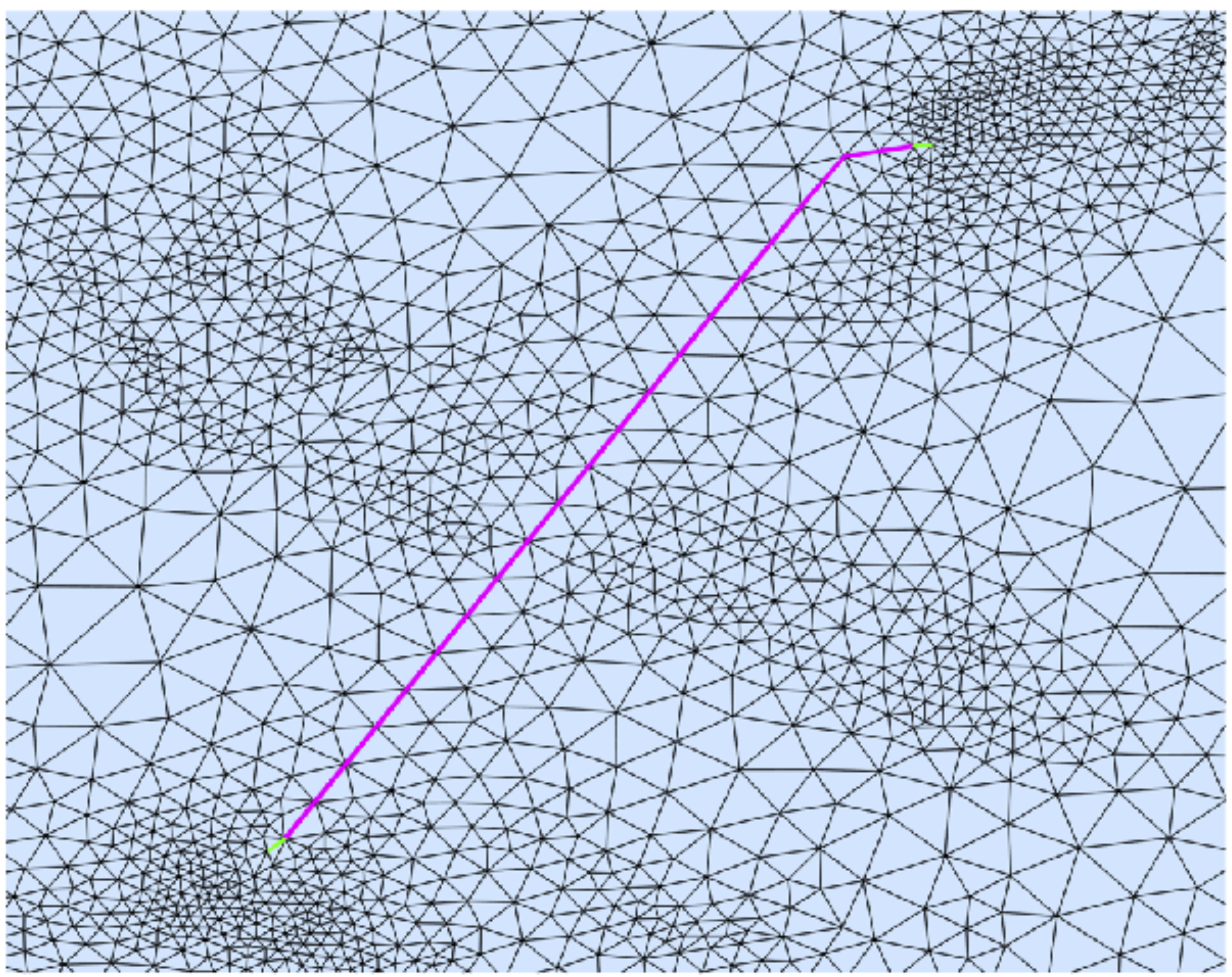


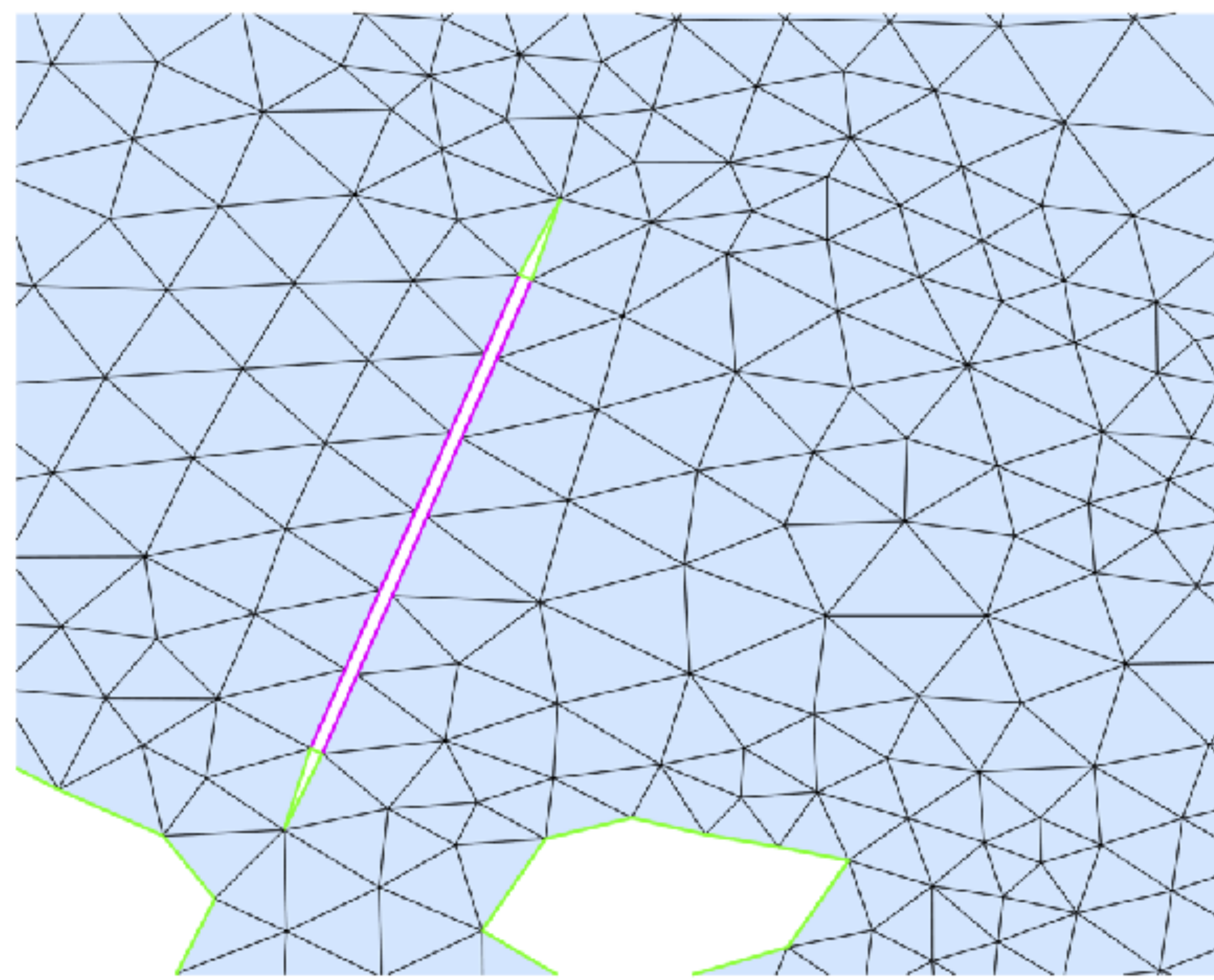



```
%% STEP 1: set mesh extents and set parameters for mesh.
bbox      = [-73.97 -73.75      % lon_min lon_max
             40.5 40.68];       % lat_min lat_max
min_el    = 15.0;                   % Minimum resolution in meters.
max_el    = 1e3;
dt        = 2;                 % Ensure mesh is stable at a 2 s timestep
grade     = 0.15;              % Mesh grade in decimal percent.
R         = 3;                 % Number of elements to resolve feature width.
%% STEP 2: specify geographical datasets and process the geographical data
%% to be used later with other OceanMesh classes...
coastline = 'PostSandyNCEI';
dem       = 'PostSandyNCEI.nc';
load weirs
gdat = geodata('shp',coastline,...
               'dem',dem,...
               'bbox',bbox,...
               'h0',min_el,...
               'weirs',weirs);
%% STEP 3: create an edge function class
fh = edgefx('geodata',gdat,...
            'fs',R,...
            'dt',dt,...
            'max_el',max_el,...
            'g',grade);
%% STEP 4: Pass your edgefx class object along with some meshing options and
% build the mesh...
mshopts = meshgen('ef',fh,'bou',gdat,'plot_on',1,'proj','utm',...
                  'dj_cutoff',1e-4);
% now build the mesh with your options and the edge function.
mshopts = mshopts.build;

m = mshopts.grd;

%% STEP 5: Manually specify open boundaries and weir crest heights
% note you will need manual human input here
m = makens(m,'outer',0) ;  % specify your elevation specified boundaries using the cursor
m = makens(m,'weirs',gdat); % make the nodestring boundary conditions

%% STEP 6: interpolate bathy and plot and save the mesh
m = interp(m,gdat,'nan','fill','mindepth',1); % interpolate bathy to the
                      % mesh with fill nan option to make sure corners get values
plot(m,'bd'); % visualize your boundaries
plot(m,'bmesh'); % plot triangulation and bathy
caxis([-10 0]) ;
save('JBAY_HR.mat','m')
```

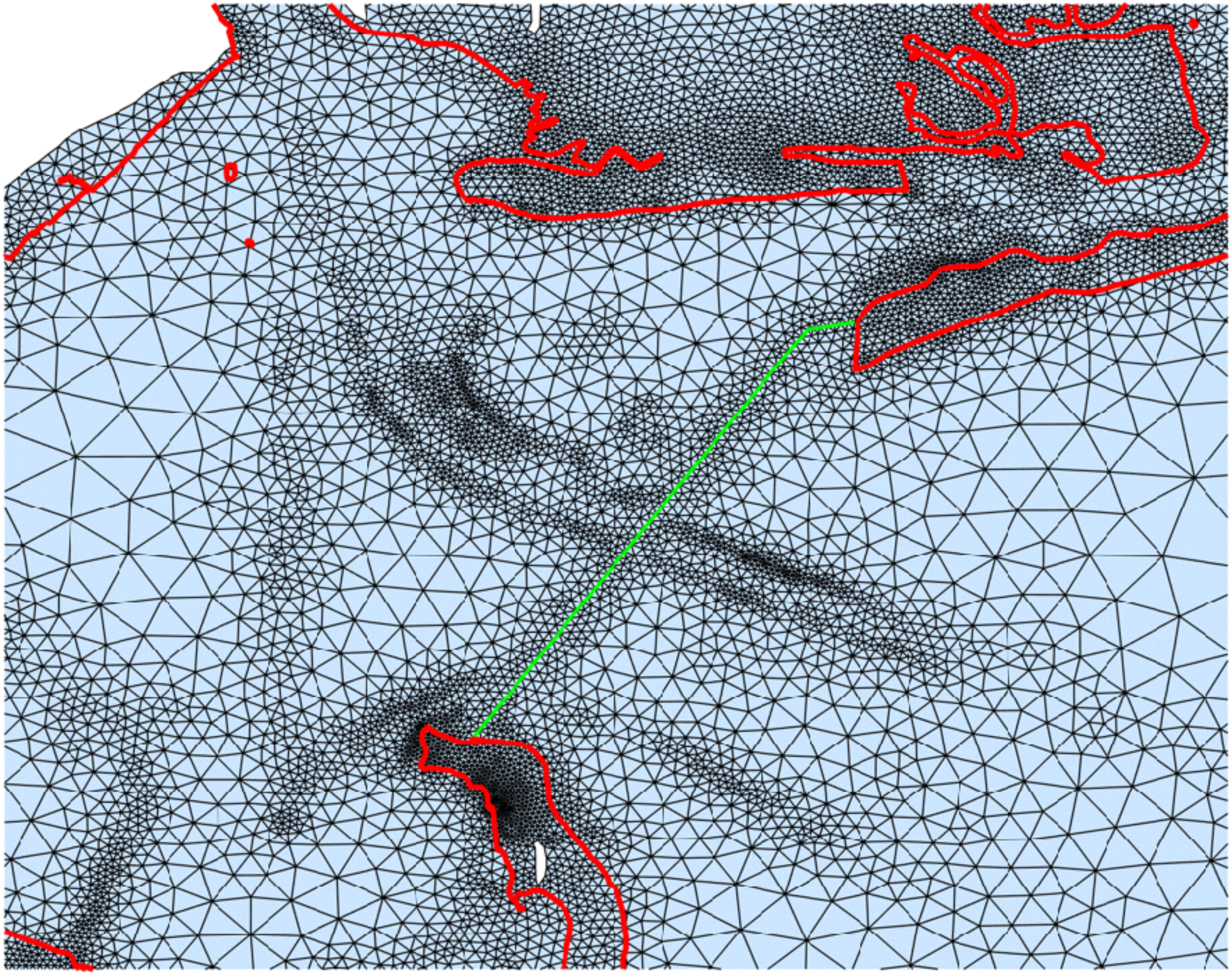
Meshing Barriers



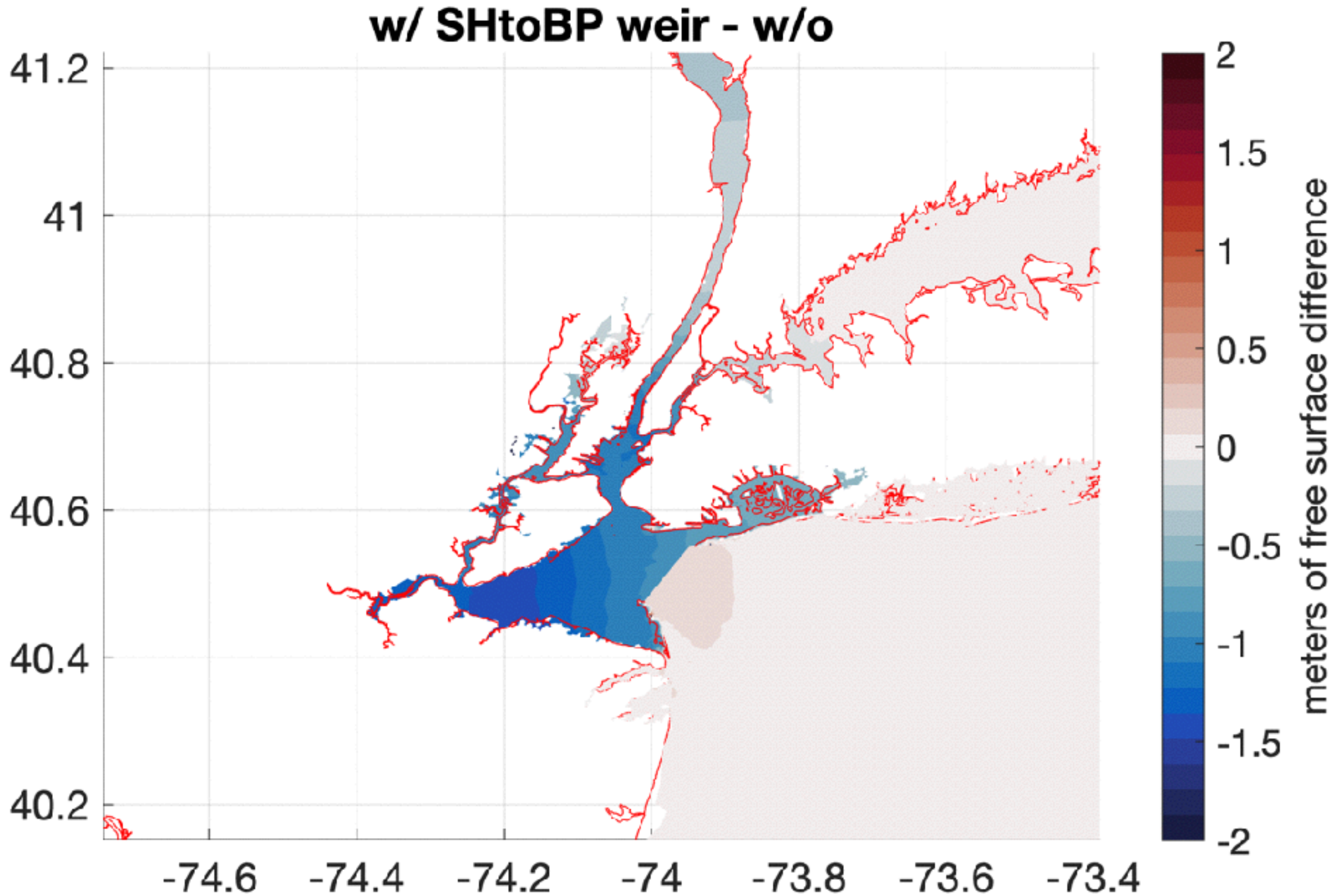




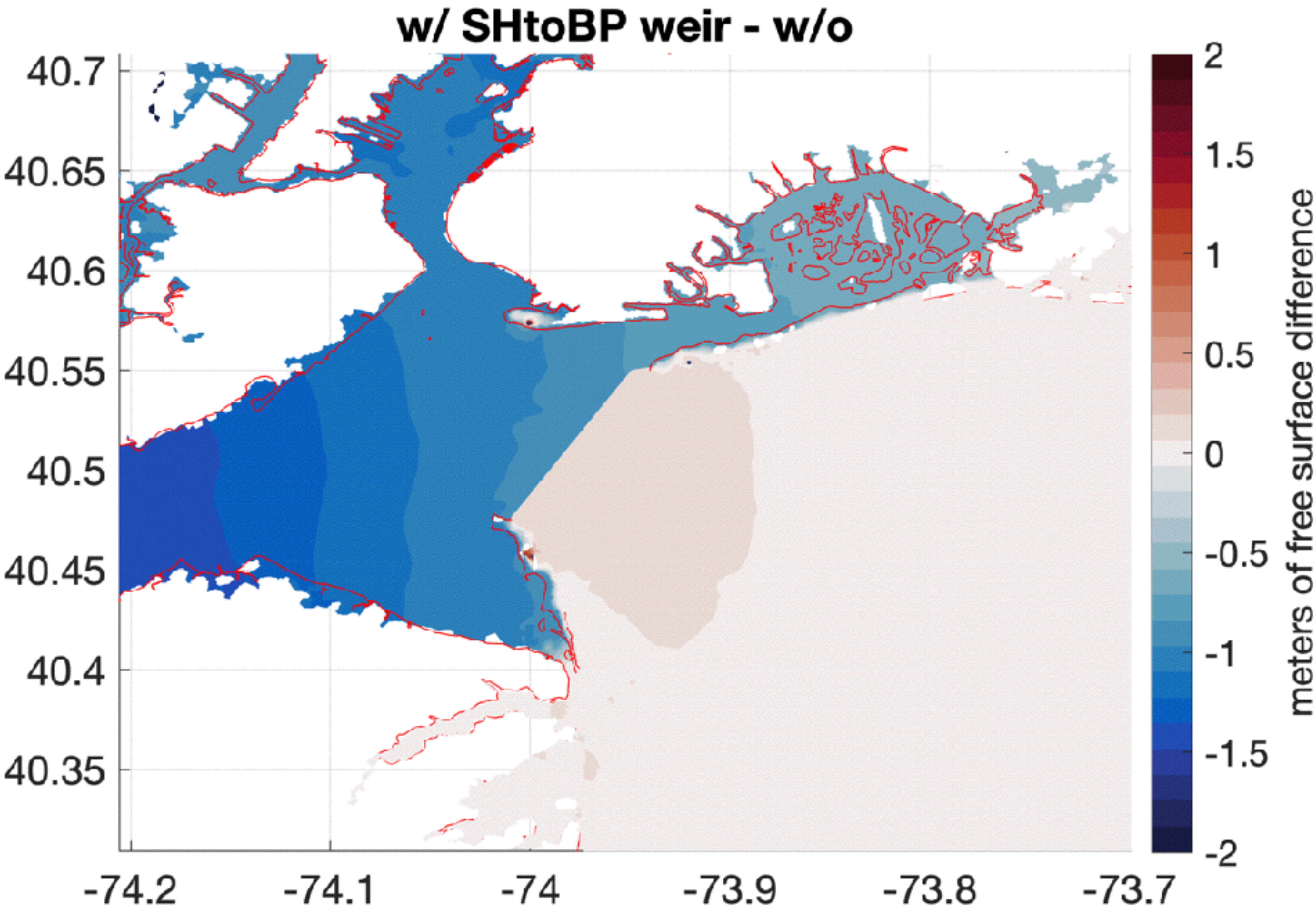
Sandy Hook to Breezy Point (SH-BP)



Maximum water levels during Hurricane Sandy



Maximum water levels during Hurricane Sandy



Conclusions

Eight step-by-step examples are here:

https://github.com/CHLNDDEV/OceanMesh2D

OceanMesh2D algorithm and operations

[1] - Roberts, K. J., Pringle, W. J., and Westerink, J. J., 2019.
OceanMesh2D 1.0: MATLAB-based software for two-dimensional unstructured mesh generation in coastal ocean modeling,
Geoscientific Model Development, 12, 1847-1868. https://doi.org/10.5194/gmd-12-1847-2019.

[2] - Roberts, K. J., Pringle, W. J, 2018.
OceanMesh2D: User guide - Precise distance-based two-dimensional automated mesh generation toolbox intended for coastal
ocean/shallow water. https://doi.org/10.13140/RG.2.2.21840.61446/2.

Mesh design using OceanMesh2D

[3] - Roberts, Keith J. Unstructured Mesh Generation and Dynamic Load Balancing for Coastal Ocean Hydrodynamic Simulation, 2019.
https://curate.nd.edu/show/4q77fr0022c

[4] - Roberts, Keith J., Pringle W.J., Westerink J. J. Contreras, M.T., Wirasaet, D., 2019.
On the automatic and a priori design of unstructured mesh resolution for coastal ocean circulation models,
Ocean Modelling, 144, 101509. https://doi.org/10.1016/j.ocemod.2019.101509

Thank you for your attention!